Decimals, percentages, proportion and ratio

Fractions, decimals, percentages, proportion and ratio
come under the umbrella of
Proportional reasoning

(making comparisons between objects or quantities in a multiplicative way)

Proportional reasoning is fundamental to real world and every day situations such as:

- Determining the better buy when 1kg of something costs £3.50 and 2.5kg of the same thing costs £4.20
- Sharing two chocolate bars between three people
- Determining if £1.04 per litre is better than a 5% discount on your bill
- Calculating the sale price when everything is 30% off

Definitions

Decimal:

- A fraction whose denominator is a power of ten and whose numerator is expressed by figures placed to the right of a decimal point
- The system of decimal numerical notation

Percentage:

- A rate, number, or amount in each hundred.
- An amount, such as an allowance or commission, that is a proportion of a larger sum of money

Proportion:

- A part, share, or number considered in comparative relation to a whole

  for example 1 in every

Fractions, decimals, and percentages are equivalent ways of expressing the same number (1/4 = 25/100 = 0.25 = 25%)

Ratio:

- The quantitative relationship between two amounts showing the number of times one value contains or is contained within the other

  for example 1 for every

Decimals

What everyone should know!

- Decimals are a form of fraction.
- Many do not actually finish, but carry on infinitely. e.g. 1/3 = 0.3 (rational / irrational numbers)
- They require a sound knowledge of place value
- How to write a fraction as a decimal fraction (i.e. 1/10 = 0.1)
- Early forms of decimal come in money (£3.11 = 311p)
- Decimals have real life applications in measures (125cm=1.25m)
- They are numbers and so can be positioned on a number line

Decimals

As these are fractions, links between these and early fractions are imperative:

$$^{1}/_{10} = 0.1$$

$$^{12}/_{100} = 0.12$$

Activities like this also link to multiplying and dividing by 10 and 100

1000	100	10	1	.	10ths	100ths

Dealing with decimals

Multiplying decimals

0.4 x 5.8

This means 0.4 of 5.8, or 4/10 of 5.8

Let's make both of these 10 times bigger by multiplying both by 10 (so ultimately multiplying by 100):

4 x 58 = 232 (by doubling and doubling again)

But… we've made the calculation 100 times bigger, so the answer is actually 100 times smaller

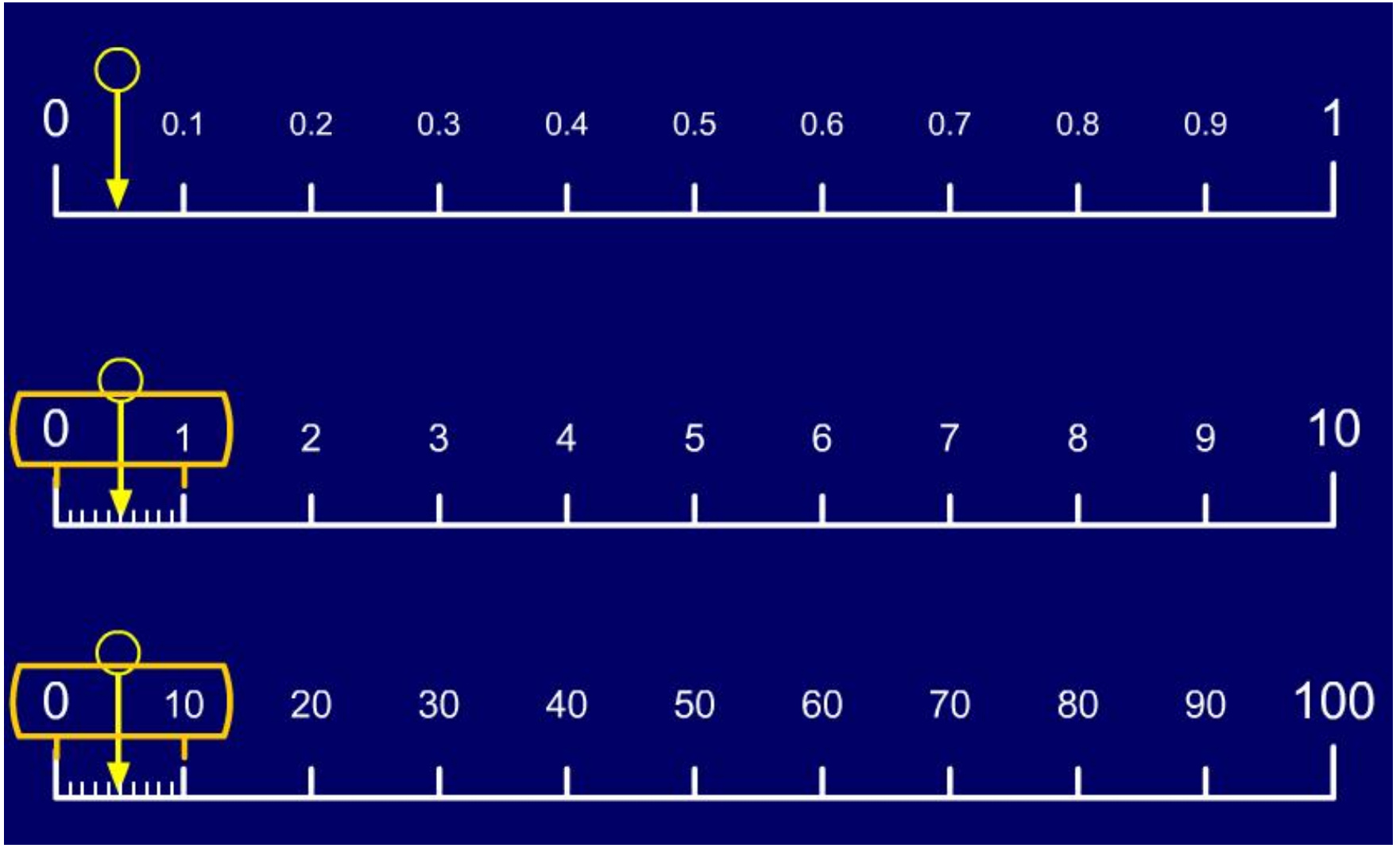
0.4 x 5.8 = 2.32

Or… you could find 1/10 and then double and double again.

We want students to be able to think flexibly and use methods that work for them!

Make a whole number…

5.4	**3.3**	**8.8**	**6.7**	**4.5**
2.6	**4.9**	**7.2**	**8.9**	**9.4**
1.7	**9.5**	**3.9**	**2.1**	**7.8**
1.3	**6.6**	**4.4**	**5.2**	**1.8**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80
90
100

Decimal problems

- If I spend £5.99 on a book and £4.25 on a toy, how much do I spend altogether?
- Sam walked to the post box 3.24km from her house. Then she walked back 2.11km and stopped. How much further did she have to walk to get home?
- Ten years ago Jess was 79.64cm tall. Now she is 83.91cm taller. How tall is she now in metres?
- Kieran has three pumpkins. They total 2.26kg in weight. One weighs 0.45kg, another 0.58kg. How much does the third weigh?
- Tanya has 7.453l of red paint, 3.041l of blue paint and 9.289l of white paint. How much paint does she have altogether?

Percentage

- A special fraction that has a denominator of 100.
- Per cent means ‘for every hundred’.

Expressing fractions as percentages

45/60 on a test

45/60 = 15/20 = 75/100 = 75%

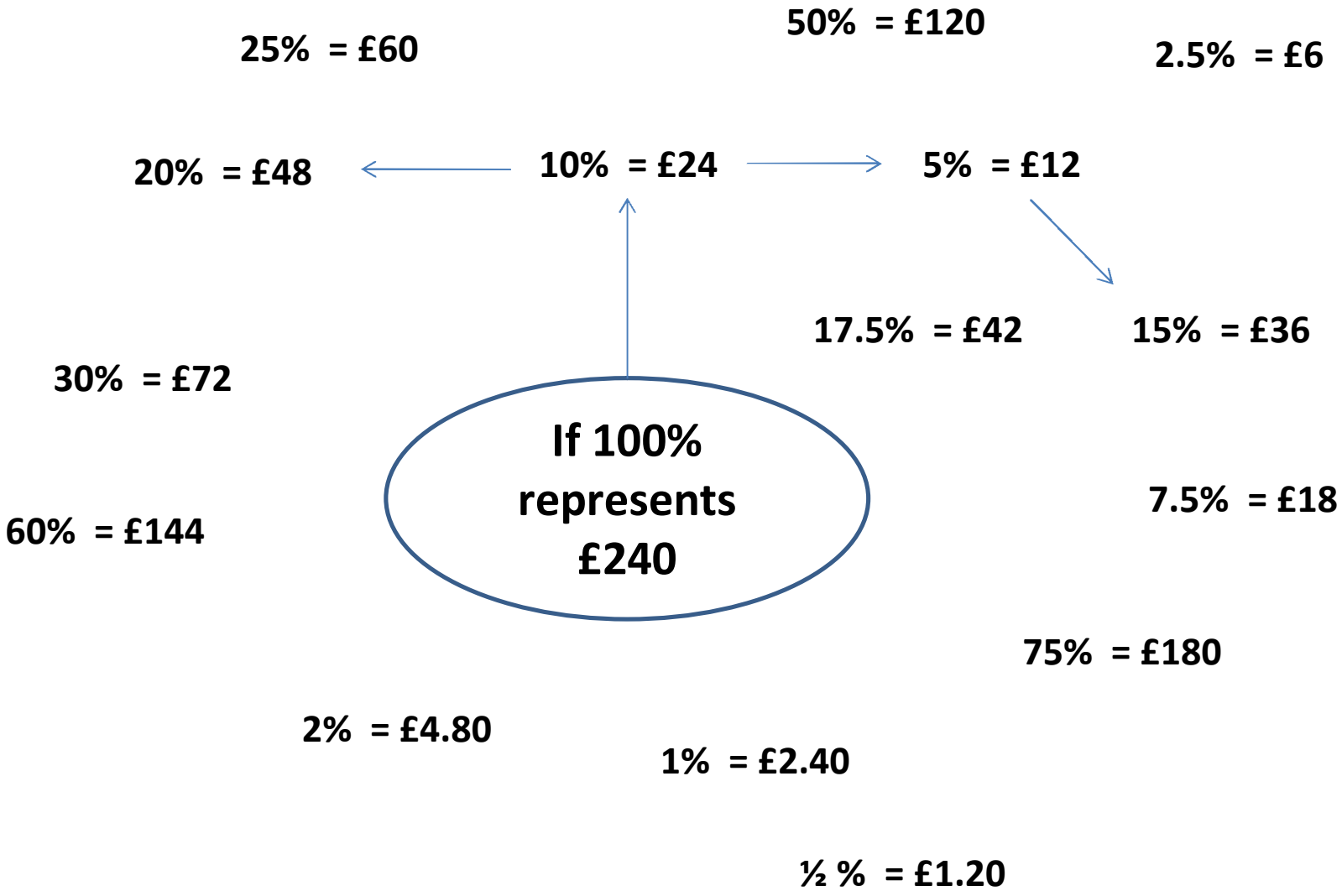
Use equivalent fractions to reduce denominator to factor of 100

Find percentages of whole number quantities

- 25% of 80 — *25/100 = ¼ so need to find ¼ of 80*
- 17 ½ % of £70 — *Find 10%, halve it for 5%, halve it again for 2.5%*

It is possible to find a percentage of any number by finding 10%, then 'play' around with it to get the percentage required.

- Find 13% of 125.
- 10% is 1/10 so find a tenth of 125 by dividing by 10 = 12.5
- 1% is 1/10 of 10% so divide 12.5 by 10 = 1.25
- Now multiply by 1% by 3 for 3% = 3.75
- Add 10% and 3% = 16.25

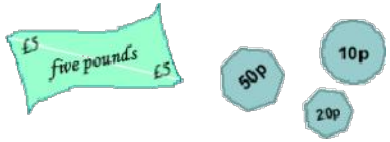


Use doubling, halving, x/÷ 10, addition and subtraction to find other percentages

Would you rather……

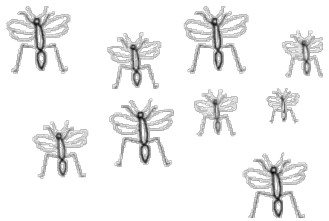
(Use doubling, halving, x/÷ 10 addition and subtraction to answer these)

…have 10% of £5 or 75% of 80p?



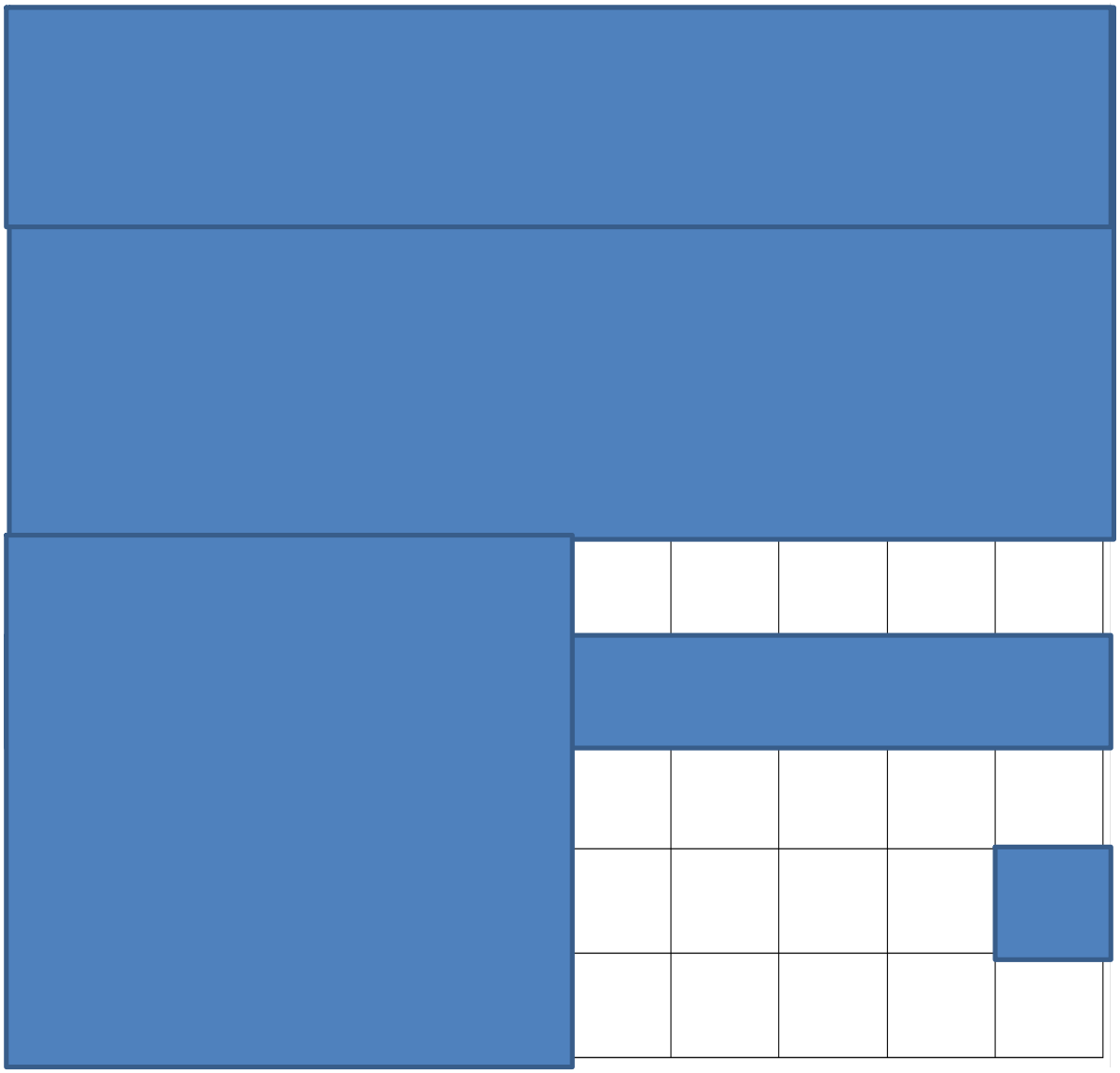
…sit in a traffic jam for 20% of 2 hours, or 40% of 1 hour and 40 minutes?

…be bitten by 15% of 120 mosquitoes or 8% of 250 mosquitoes?



100% number square

Ways to link fractions, decimals and percentages:



Percentage problems

- There are 25 students in a class. 40% of them are boys. How many are girls?
- Bobby has £800 to spend on a scooter. He only spends 80%. How much does he have left?
- Abigail ate 80% of a box of chocolates which contained 120 chocolates. How many did she eat?
- Sally weighed 84kg. She wants to loose 14% of that weight. How much does she want to weigh?
- The pop star Normandy Shields made £1 000 000 from her first hit. She paid 10% to her agent, 20% to her manager and spent 80% of what was left on clothes. She put the rest in the bank. How much did she put in the bank?

Ratio

The comparison of one quantity to another:

- The ratio of red to blue squares is 2:5.
- For every two red squares, there are five blue ones

Proportion

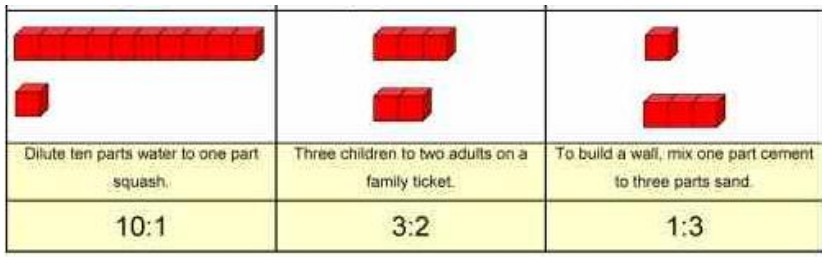
Equality of two ratios:

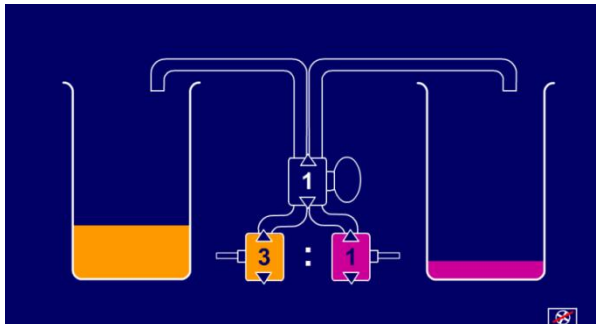
- In every seven, 2 will be red, and 5 will be blue (2/7 are red, 5/7 are blue)

10 sweets: 2 toffees, 8 chocolates

Ratio: the amount of one thing compared to another, one for every 2:8

Proportion: equality of two ratios, one out of every 2/10, 1/5, 20% toffees, 8/10, 4/5, 80% chocolates





Mixing drinks: 3 parts tonic to 1 part gin
3:1
¼ of drink is gin, ¾ tonic

Proportional Problems

- At a football club there are 4 boys for every 3 girls. There are 27 girls. How many boys are there?
- The exchange rate is $1.50 for every pound. How many dollars do you get for £12?
- Peter has some sweets. Four are toffees the rest are mints. The ratio of toffees to mints is 1:3. How many sweets does Peter have?
- There are 180 biscuits in a tin. There are 3 custard cream for every 9 bourbons. How many custard creams are there in the tin?
- Petrol costs £1.18 per litre. There is a special offer: £1.08 per litre or 5% off total bill. You buy 40 litres. Which is the best offer?