The Early Years

27.03.13

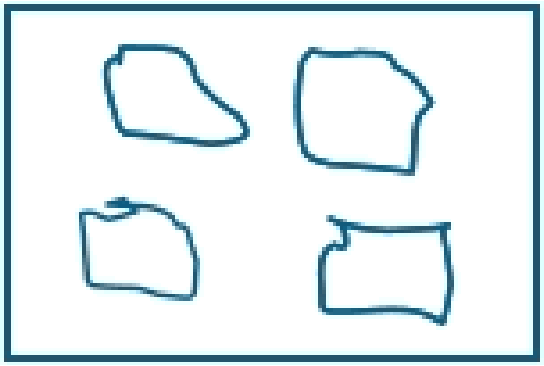
Where it all begins……

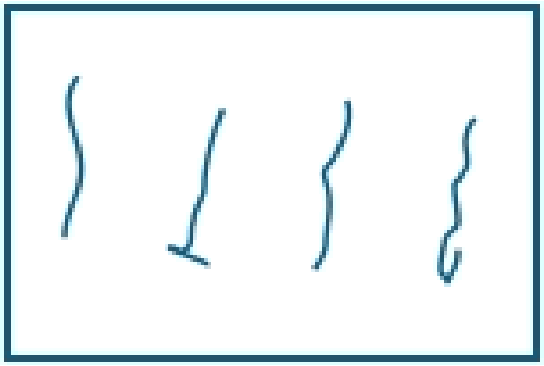
- Counting:
  - Stable order principle
  - One to one principle
  - Cardinal principal
  - Order irrelevance principal
  - Abstraction principle

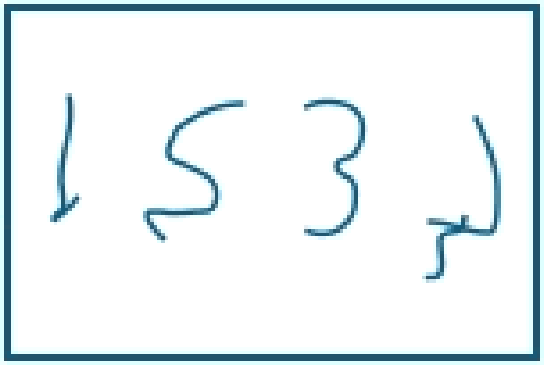
- One-ness of one etc.

- Place value
  - Straws
  - Exchange

Representing bricks





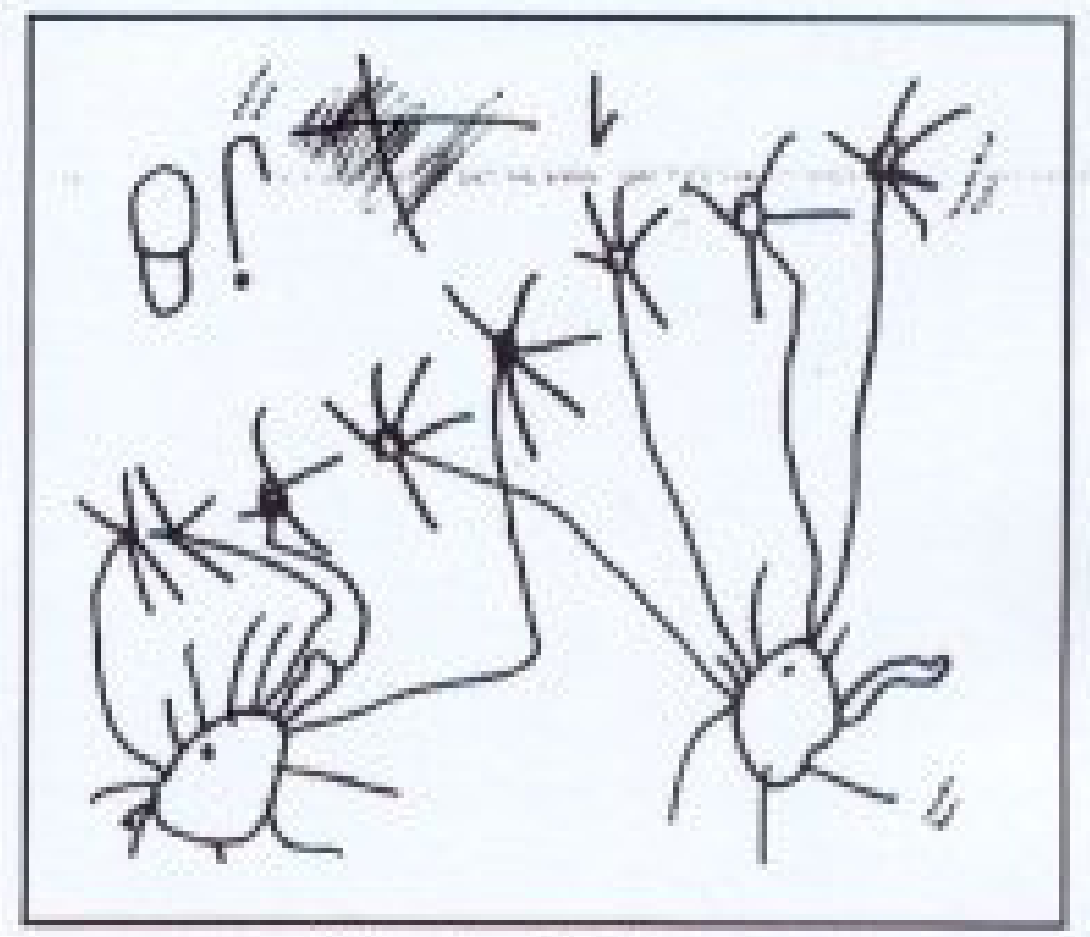


pictographic responses involved an attempt to represent the bricks in some way, as well as representing their actual numerosity
iconic responses similarly involved one-to-one correspondence.
symbolic responses involved the use of conventional symbols such as numerals
idiosyncratic responses were those that are not obvious

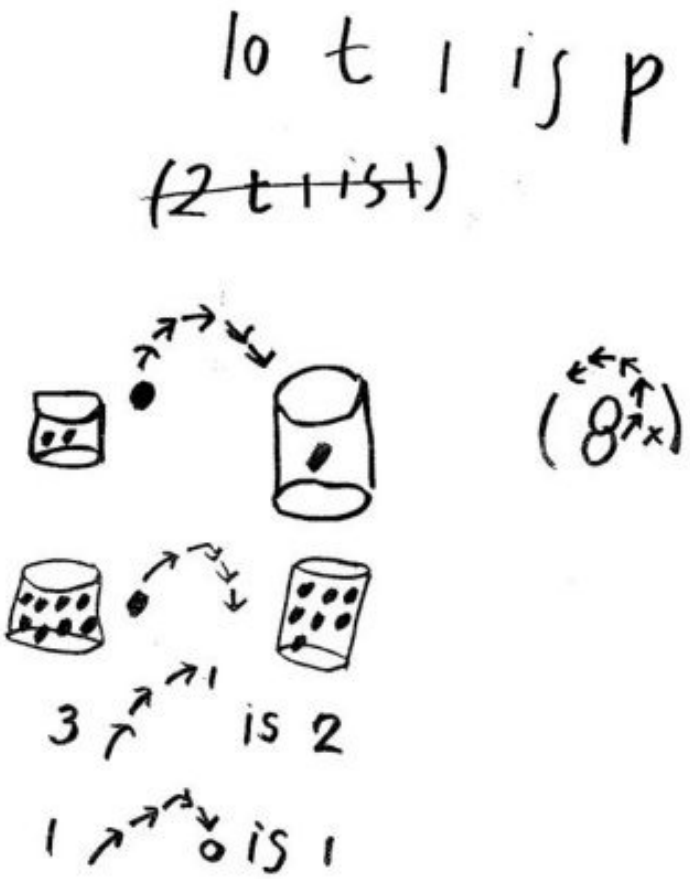
Kamrin:

Can 8 be divided into 2?

5yrs 7months

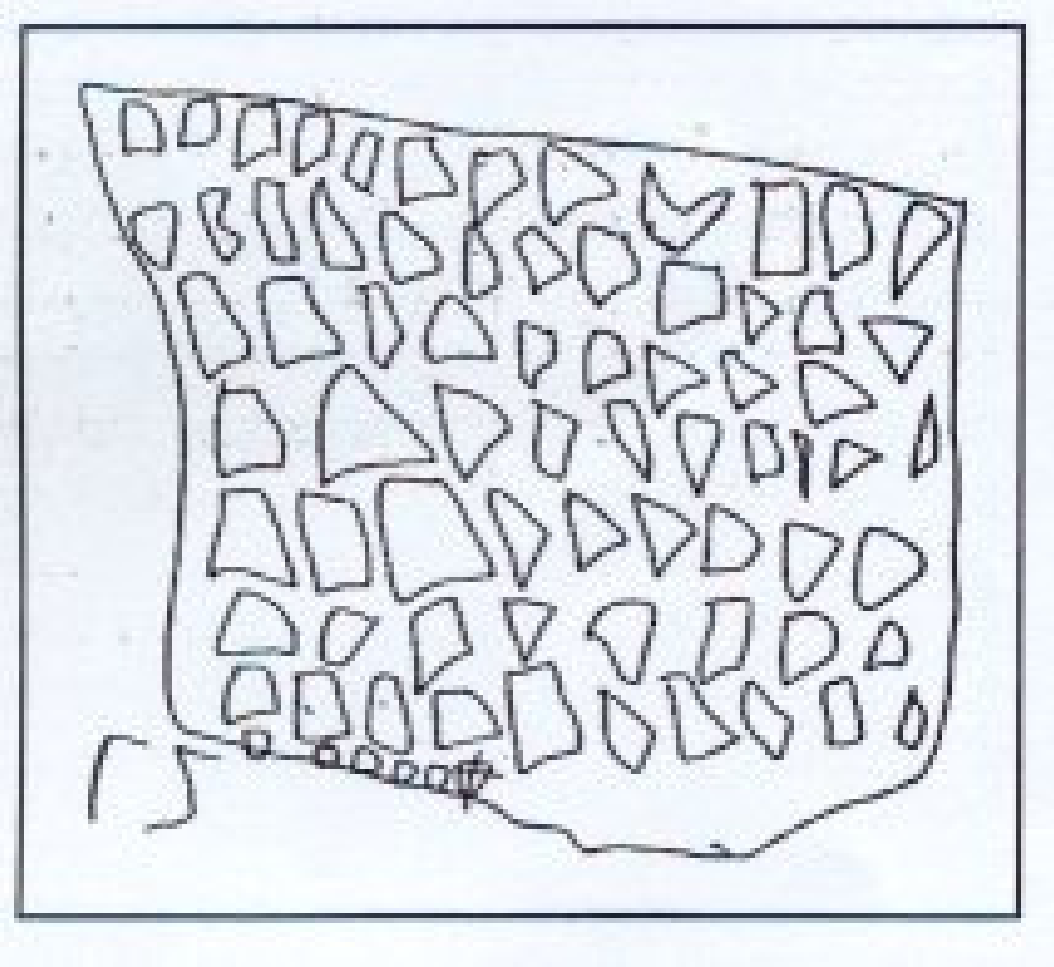


Barney
Subtracting beans
5 yrs 7 months



Frances: The train

6yrs 1month



The Early Years/Foundation Stage Early Learning Goals

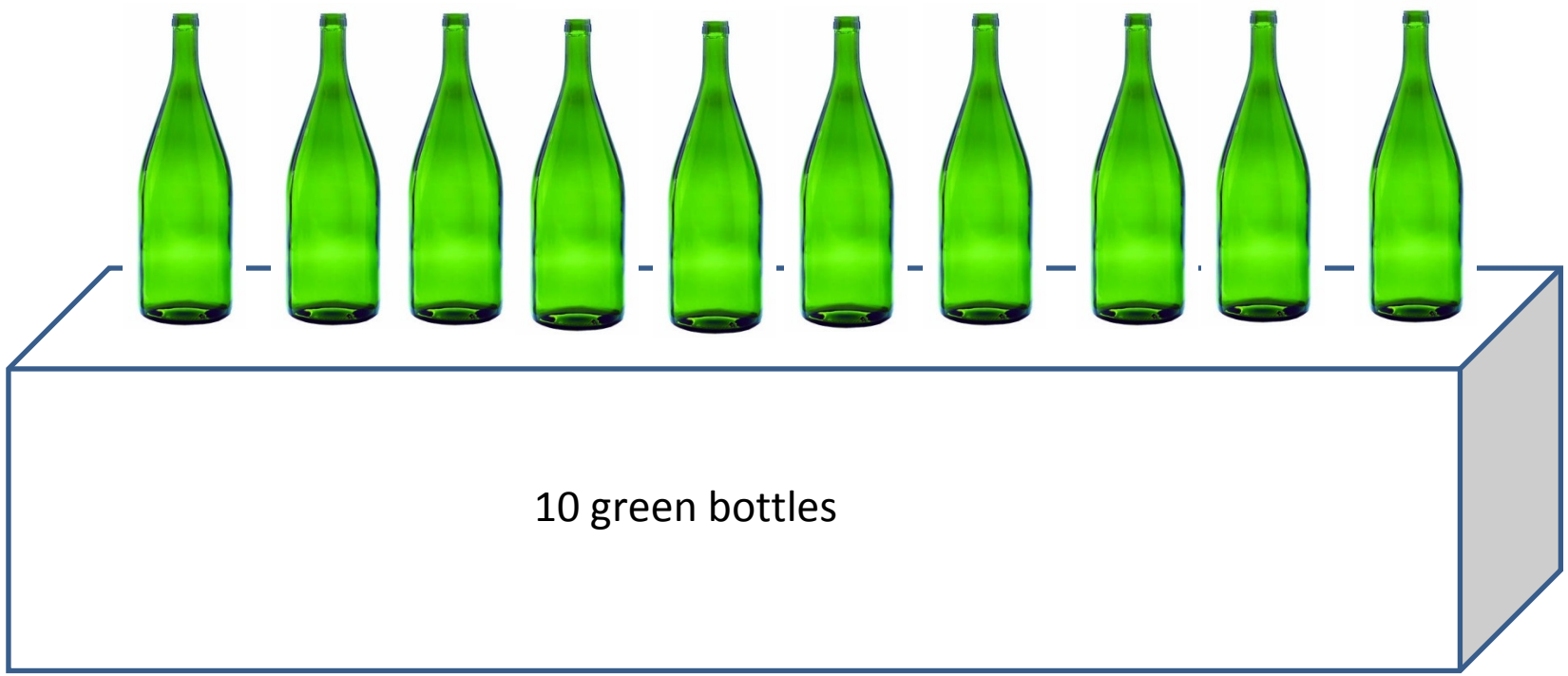
Mathematics involves providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measures.

This is what the children should be able to do by the time they leave Reception:

Numbers: children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number.

Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Shape, space and measures: children use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects and to solve problems. They recognise, create and describe patterns. They explore characteristics of everyday objects and shapes and use mathematical language to describe them.



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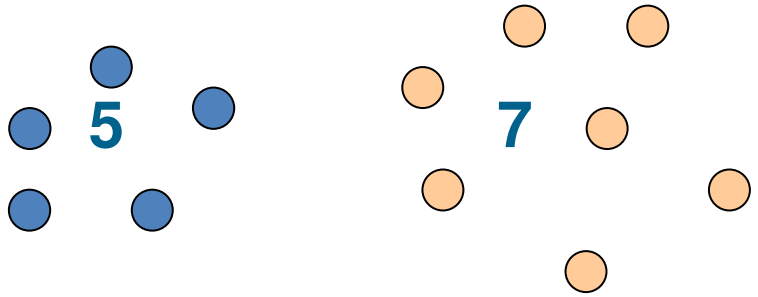
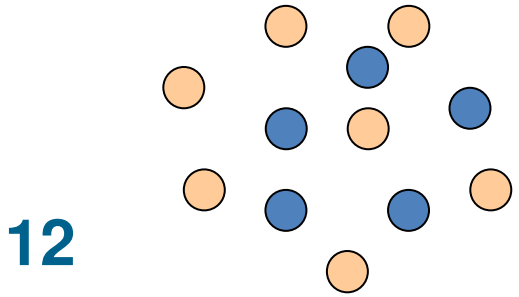
Models for addition

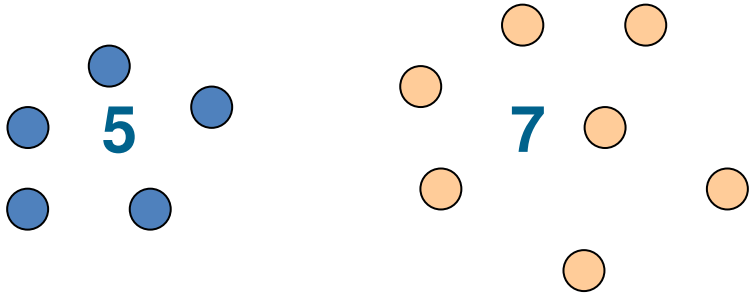
Combining two sets of objects (aggregation)



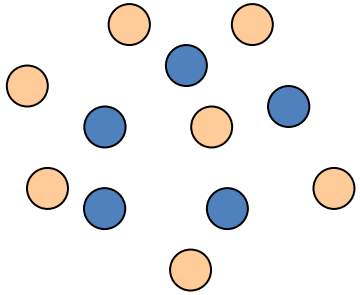


Issue:
Tend to count one set, count the other and then count all.

Adding on to a set (augmentation)

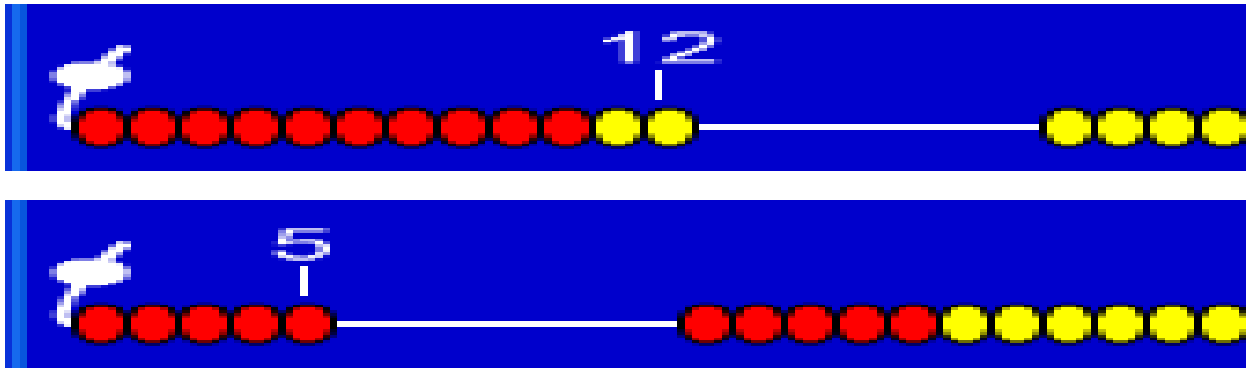


12

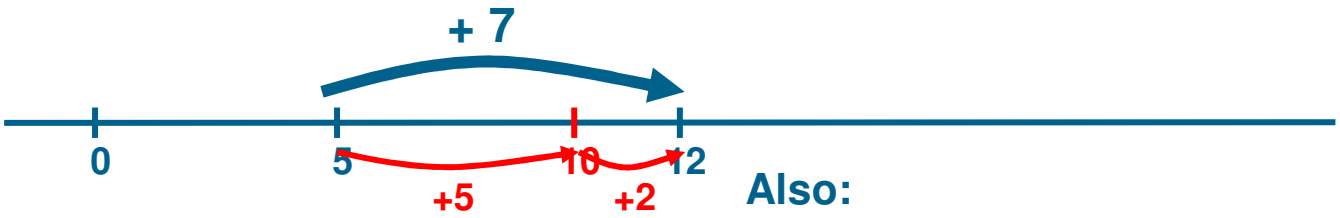


Issue:
Requires fluency with counting from any number.

Counting on with a bead bar/number line



Issues:
Bead bar is a useful bridge from cardinal to ordinal.
Number line helps to stop counting all



Also:
Bead bar and number line (showing 10s) encourages use of number bonds and place value for added efficiency.

More than single digits?

Partitioning

48 + 33

40 8 30 3 = 70+11= 81

48
+ 33
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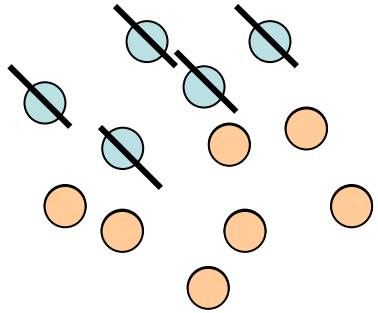
Sequencing

48 + 30 + 3 = 78 +3 = 81

Models for subtraction

12 - 5 = 7

Removing items from a set (reduction or take-away)

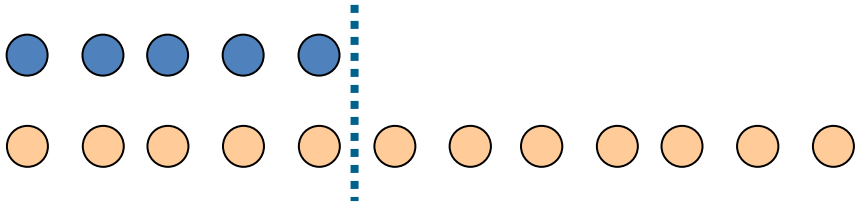


Issue:
Relies on 'counting all' again.

Models for subtraction

12 - 5 = 7

Comparing two sets (comparison or difference)

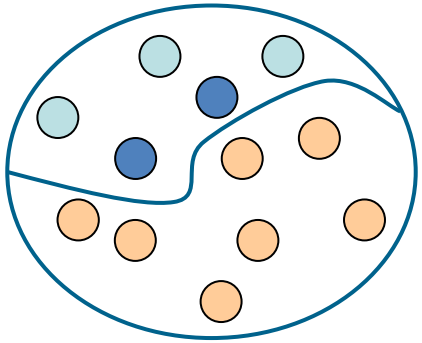


Issue:
Useful when two numbers are 'close together', where 'take-away' image can be cumbersome

Models for subtraction

12 - 5 = 7

Seeing one set as partitioned

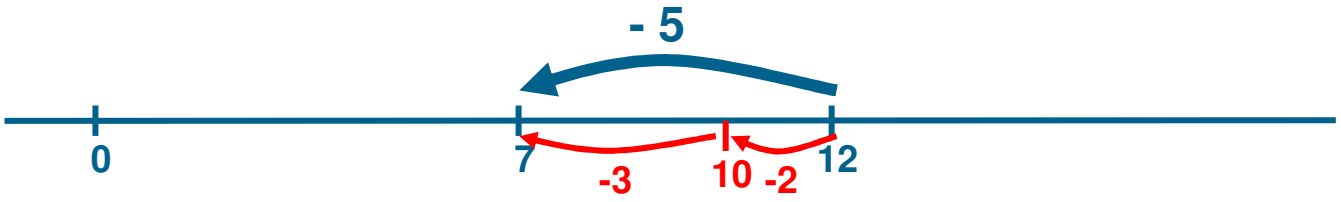


Seeing 12 as made up of 5 and 7

Issue:
Helps to see the related calculations; 5+7=12, 7+5=12, 12-7 = 5 and 12-5=7 as all in the same diagram

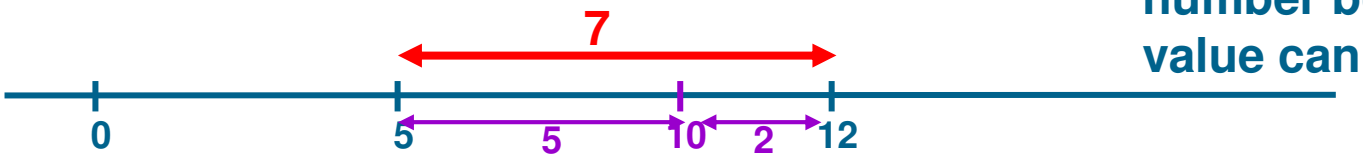
Models for subtraction

Counting back on a number line



Number line helps to stop 'counting all'.
Knowledge of place value and number bonds can support more efficient calculating

Finding the difference on a number line



Useful when two numbers are 'close together', use of number bonds and place value can help.

How can you help at home?

- Maths in the kitchen
- Maths in the bathroom
- Counting games and rhymes - these use counting skills
- Use dice - subitising
- Look at numbers in the environment
- Outside games like catch
- Tidying up games
- Making up problems
- All the things we have thought about this afternoon!

Place value, Addition and Subtraction

27.03.13

Key thresholds in mathematical development in arithmetic

- KS1 entry: conservation and counting
- KS2 entry: addition/subtraction, number bonds to 20, place value
- KS3 entry: multiplication/division, multiplication tables

Where it all begins……

- Counting:
  - Stable order principle
  - One to one principle
  - Cardinal principal
  - Order irrelevance principal
  - Abstraction principle

- Place value
  - Straws
  - Exchange

Try this with place value counters or Dienes:

1. Throw the dice
2. Count that number of ones
3. Keep going exchanging every time you make a 10
4. First person to make 50

Try this with place value counters or Dienes:

1. Collect 50 in tens counters or Dienes
2. Throw the dice
3. Take that number away
4. Keep going exchanging 10s to 1s when you need to
5. First person to get to 0 wins!

Mathematical Proficiency

- Mathematical proficiency requires a focus on core knowledge and procedural fluency so that pupils can carry out mathematical procedures flexibly, accurately, consistently, efficiently, and appropriately. Procedures and understanding are developed in tandem.

Arithmetic Proficiency:
achieving fluency in calculating with understanding

- ... an appreciation of number and number operations, which enables mental calculations and written procedures to be performed efficiently, fluently and accurately.

Written calculation:

concrete to visual to abstract, using manipulatives to ‘open the door’ to conceptual understanding

Common errors and misconceptions

$$\begin{array}{r} 35 \\ +\,47 \\ \hline {\scriptstyle 2}\,91 \end{array}$$

$$\begin{array}{r} 35 \\ \times\ 3 \\ \hline 915 \end{array}$$

$$\begin{array}{r} 45 \\ -\,37 \\ \hline 12 \end{array}$$

$$2\overline{)\,216}\ \ 18$$

$$\begin{array}{r} 248 \\ \times\,25 \\ \hline 1240 \\ 496 \\ \hline \end{array}$$

Pupil forgot to ‘put a nought’ here because there already was one here.

A sledgehammer to crack a nut

$$\begin{array}{r} {\scriptstyle 0}\,{\scriptstyle 9}\,{\scriptstyle 9}\,{\scriptstyle 1} \\ 1\,0\,0\,0 \\ -\quad\;\; 7 \\ \hline 9\,9\,3 \end{array}$$

$$\begin{array}{r} {\scriptstyle 0}\,{\scriptstyle 1} \\ 1\,6 \\ -\;\; 9 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 97 \\ \times\; 100 \\ \hline 00 \\ 000 \\ 9700 \\ \hline 9700 \end{array}$$

$$\begin{array}{r} 0\,8 \\ \hline 7\overline{)\,{\scriptstyle 0}\,{\scriptstyle 5}\,6} \end{array}$$

How would you solve these?

- 25 + 42
- 25 + 27
- 25 + 49
- 145 +127

- 67 - 45
- 67 – 59
- 178 - 99
- 3241 - 2167

Well known mental calculation strategies

- Partition and recombine
- Doubles and near doubles
- Use number pairs to 10 and 100
- Adding near multiples of ten and adjusting
- Using patterns of similar calculations
- Using known number facts
- Bridging though ten, hundred, tenth

Well known mental calculation strategies

- Use relationships between operations
- Counting on
- x4 by doubling and doubling again
- x5 by x10 and halving
- x20 by x10 and doubling

Addition

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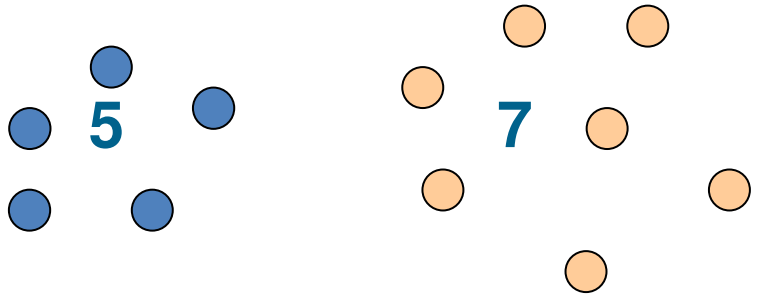

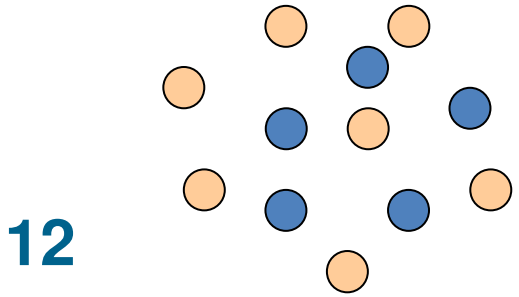

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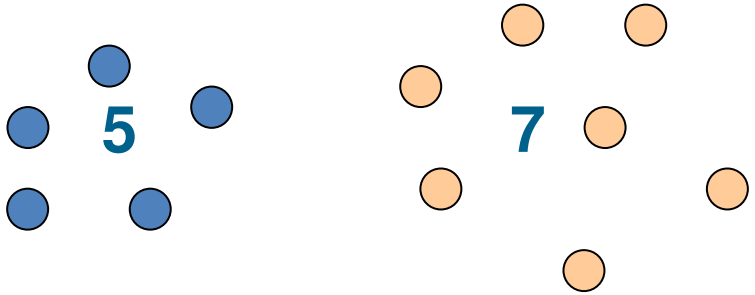

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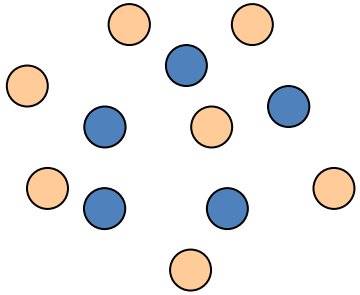

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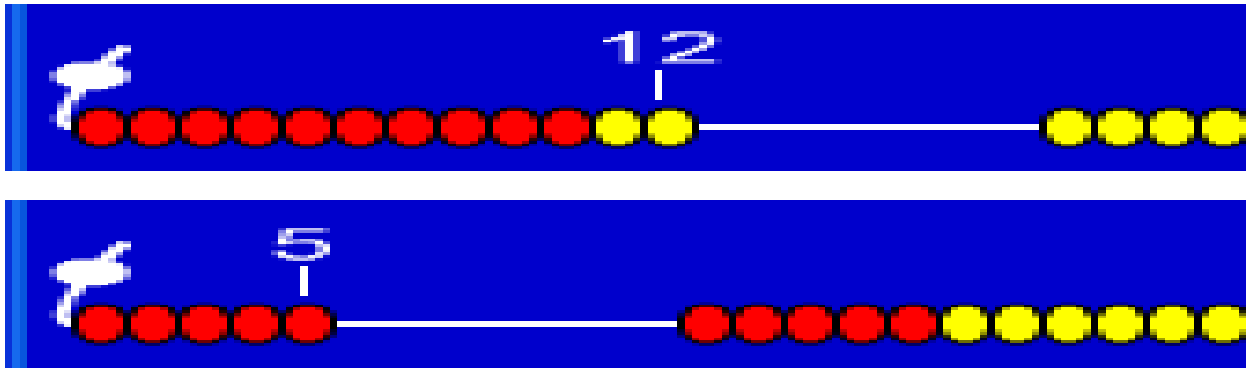

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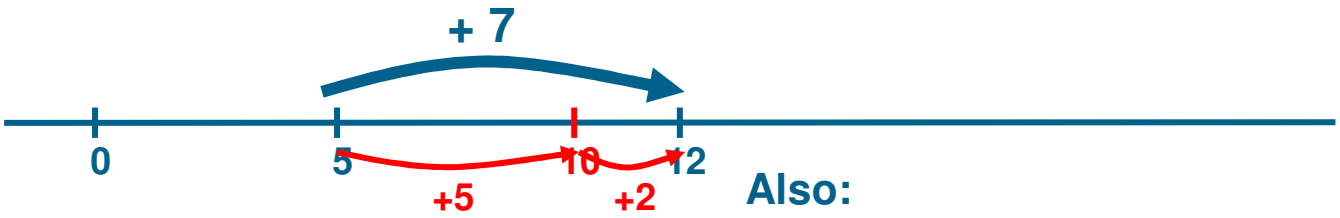

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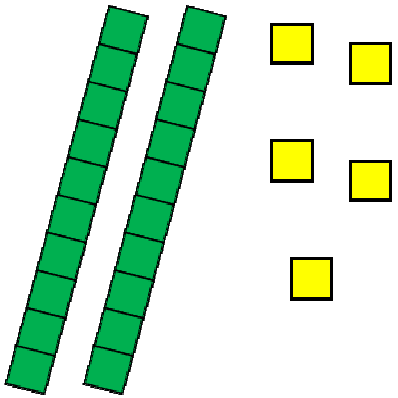
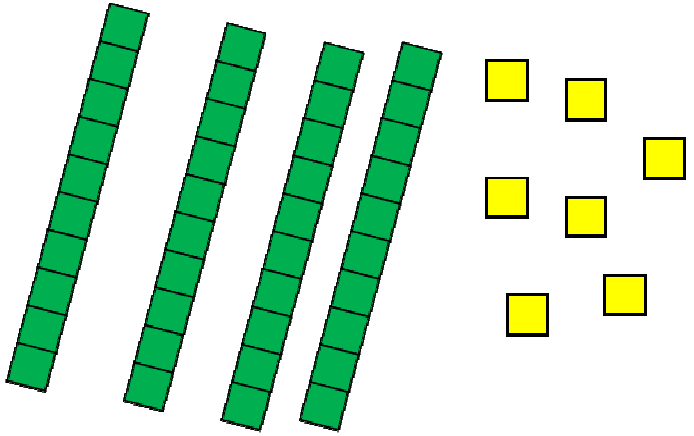
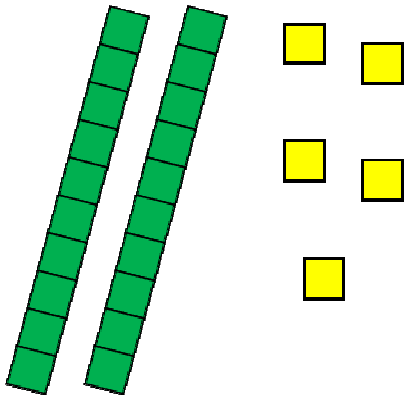
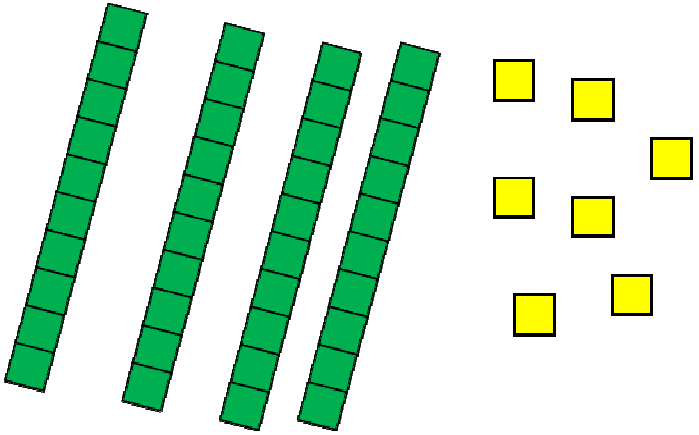
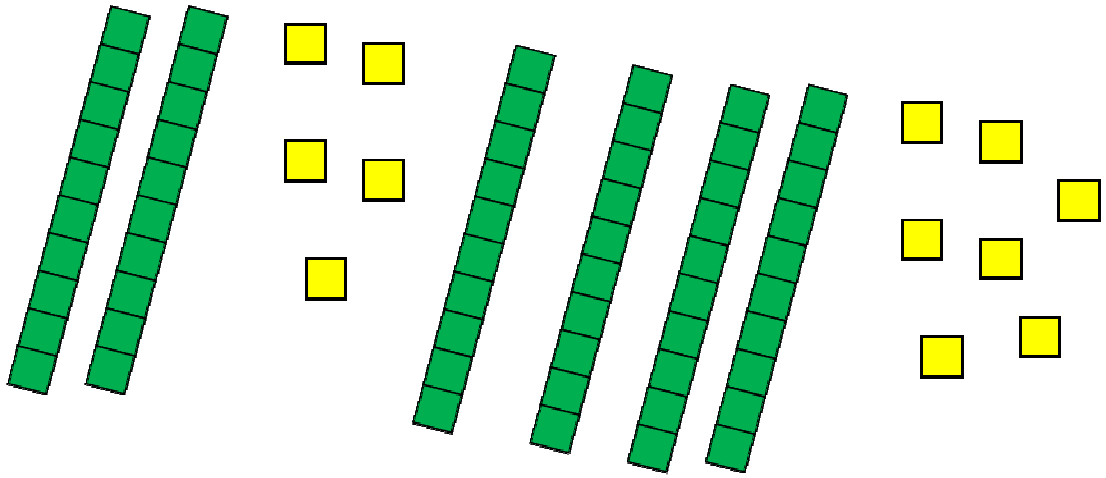
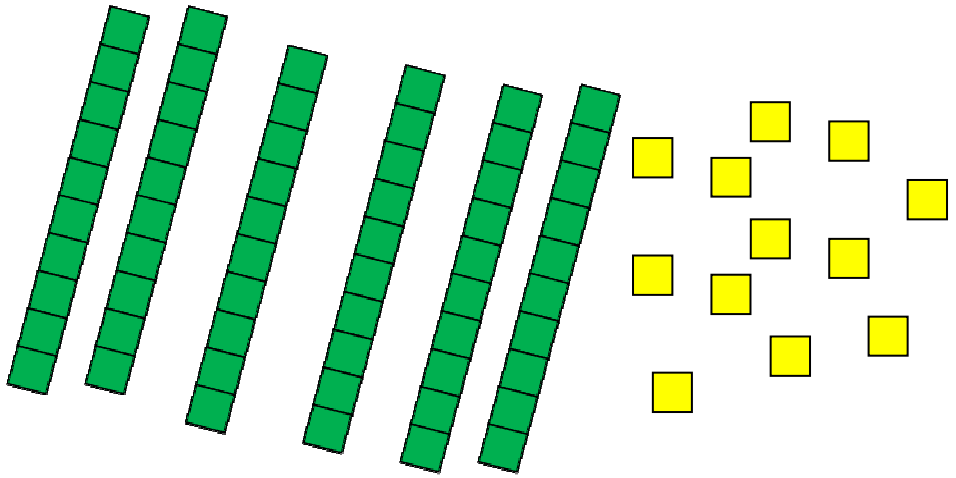


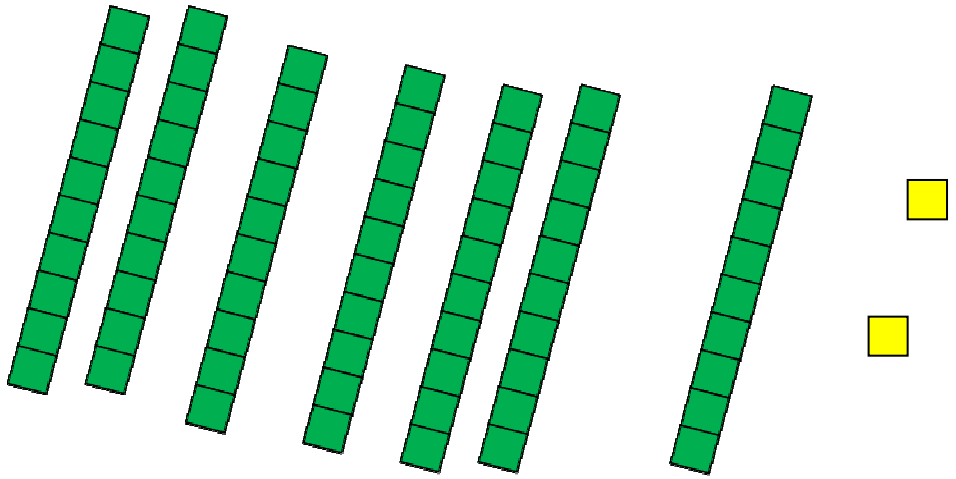




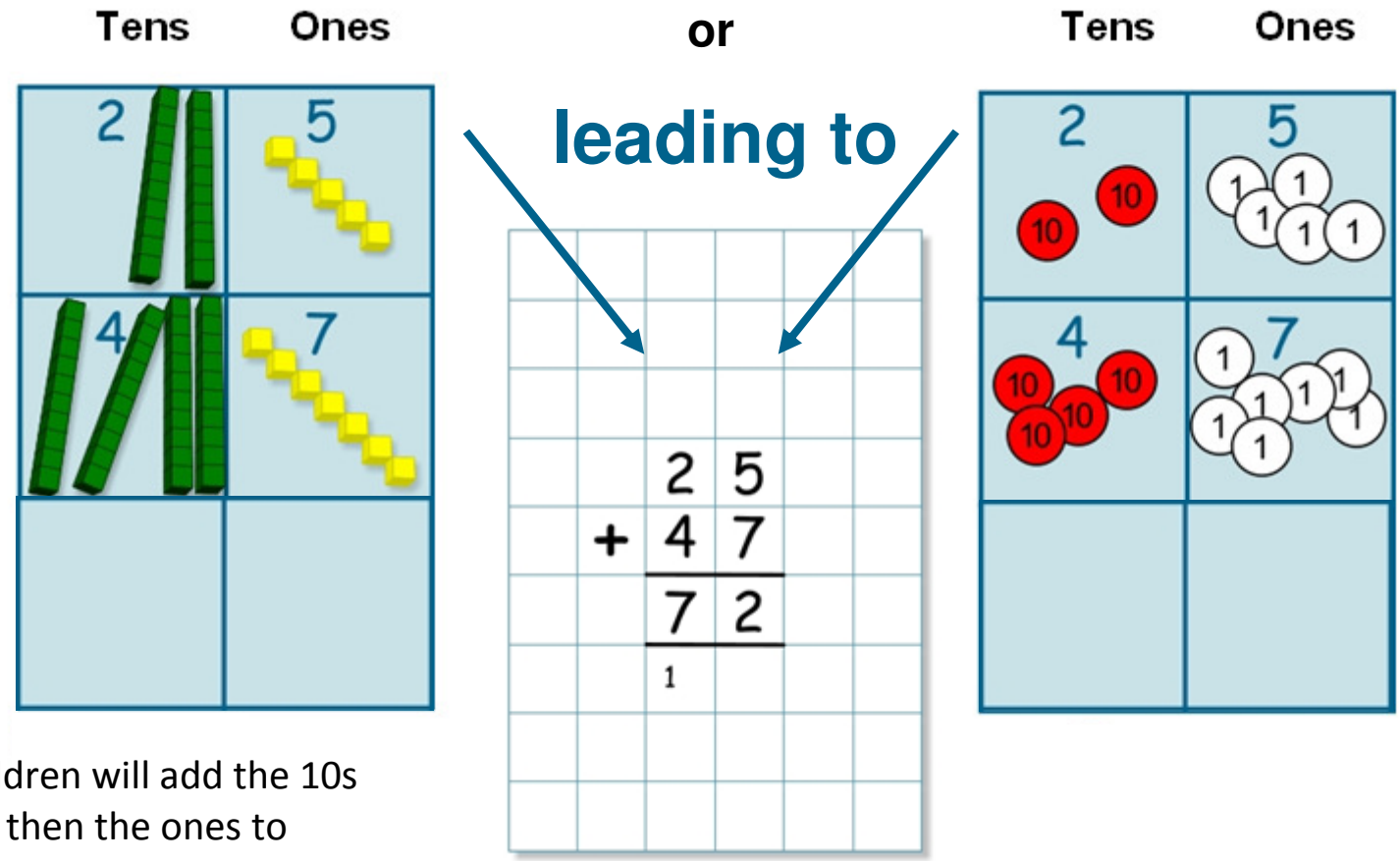






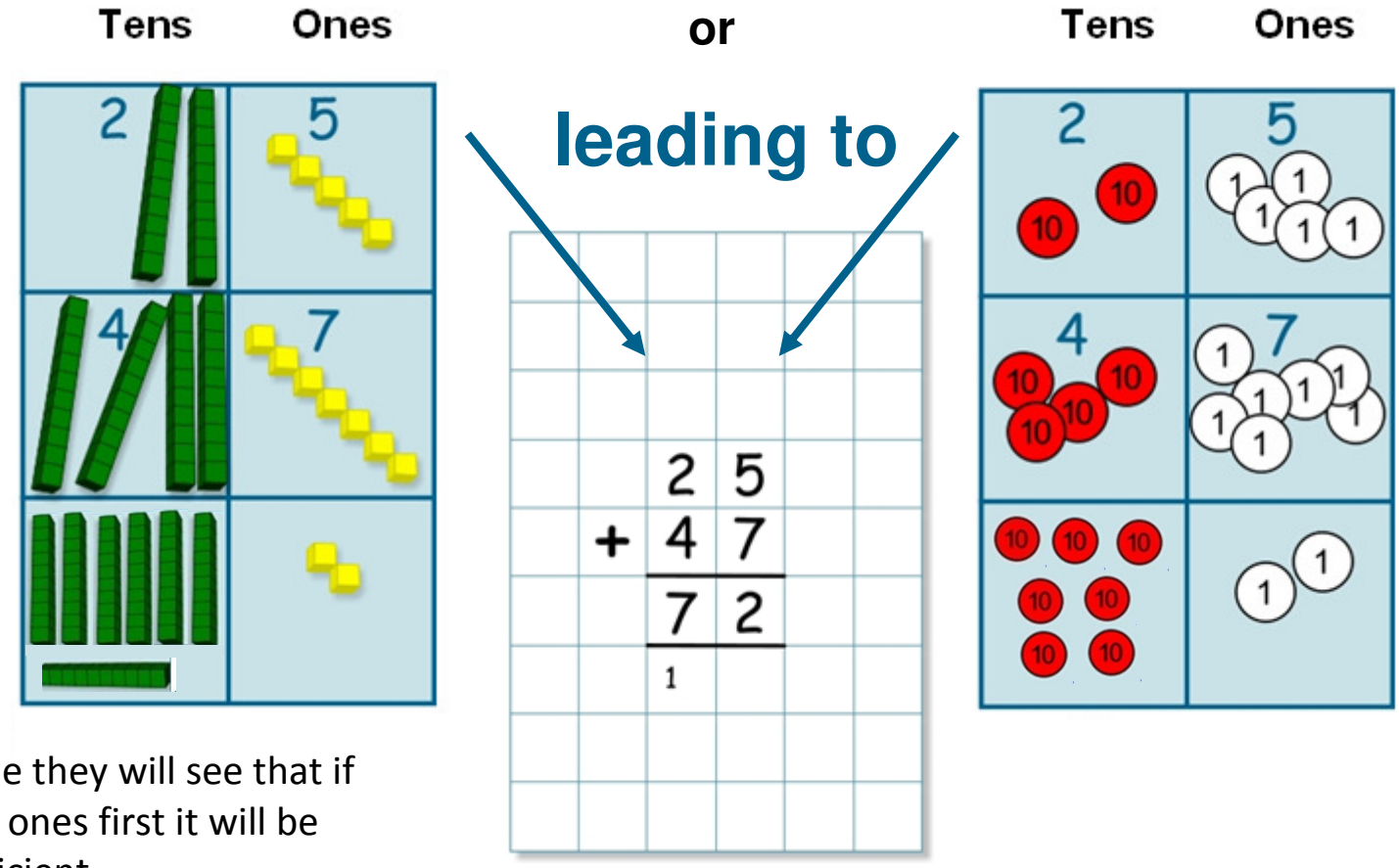


Compacted



Most children will add the 10s first and then the ones to make 60 + 12. They then adjust the ones to make 72.

Compacted



After time they will see that if they add ones first it will be more efficient.

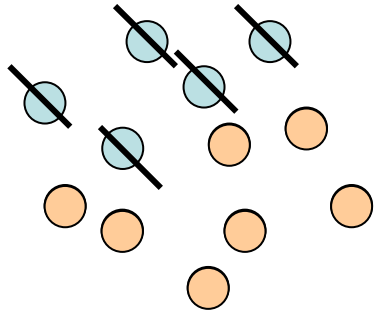
What is the same and what is different about these models?

Subtraction

Models for subtraction

12 - 5 = 7

Removing items from a set (reduction or take-away)



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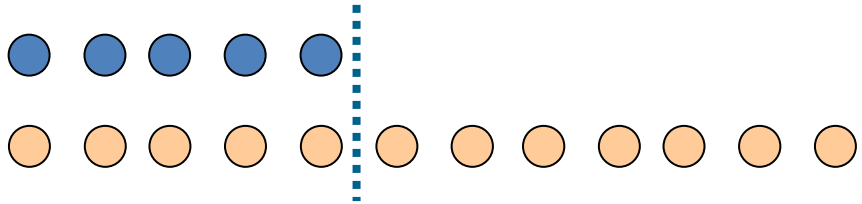

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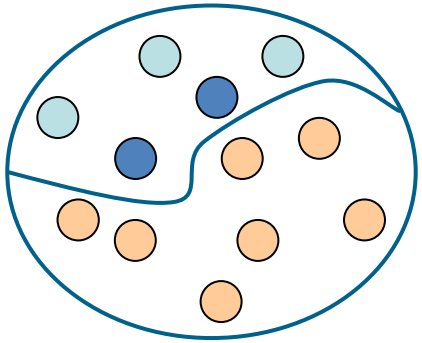

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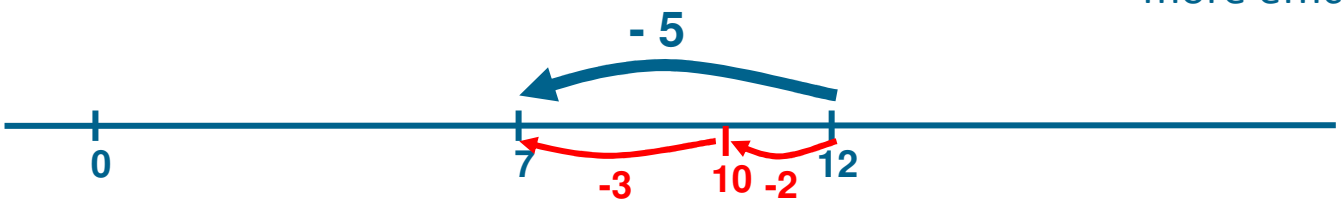
Models for subtraction

Counting back on a number line



Number line helps to stop 'counting all'.
Knowledge of place value and number bonds can support more efficient calculating

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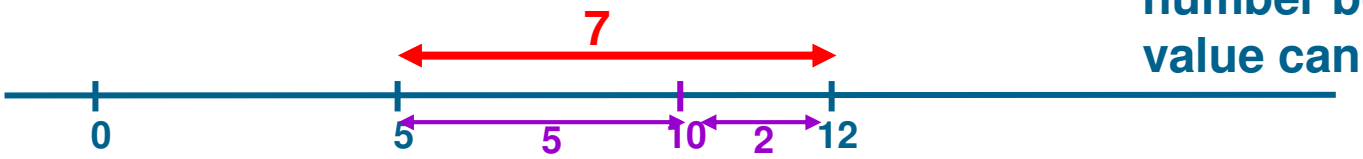

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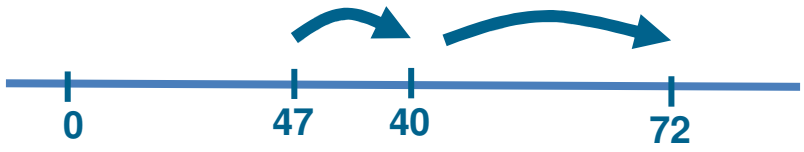
More than single digits?

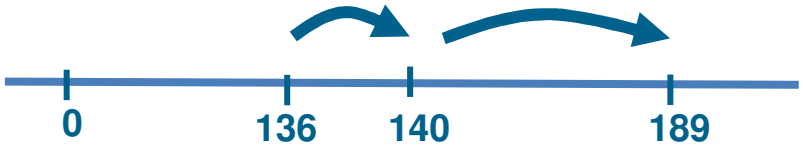
Number line, making the largest jump you can: 72 - 47



Sequencing: 72 – 47

72 - 40 – 7 = 32 – 7 = 25

Number line, making the largest jump you can: 189 - 136

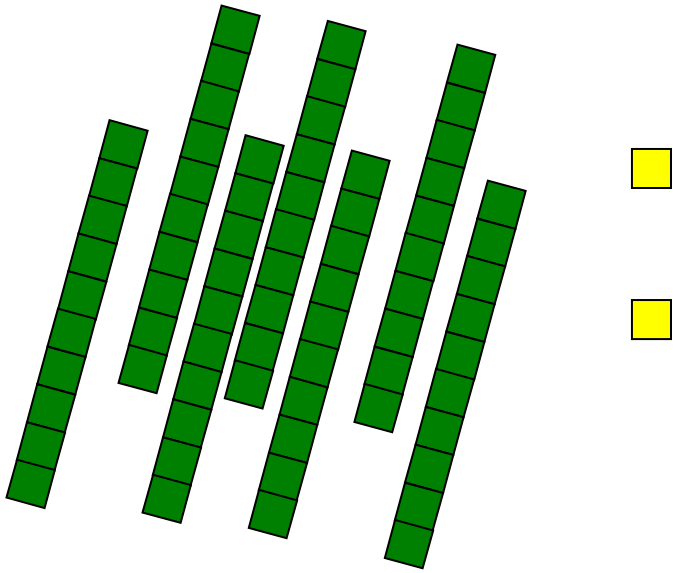


Sequencing: 189 – 136

189 - 130 – 6 = 59 – 6 = 53

More than single digits?

72 - 47



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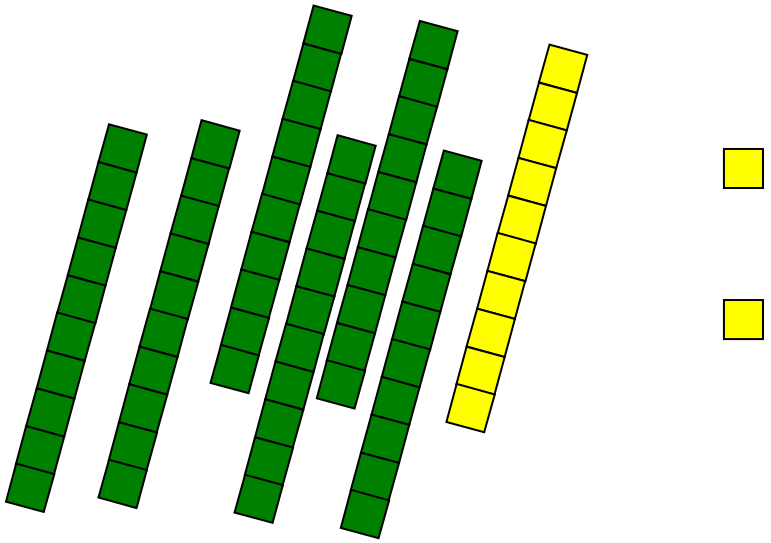
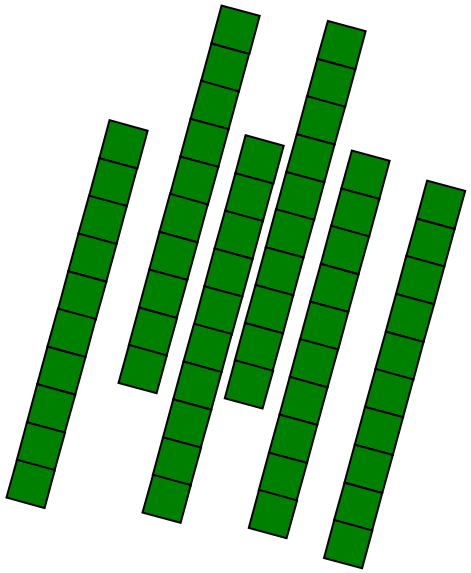
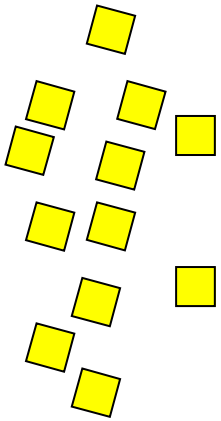


72 - 47

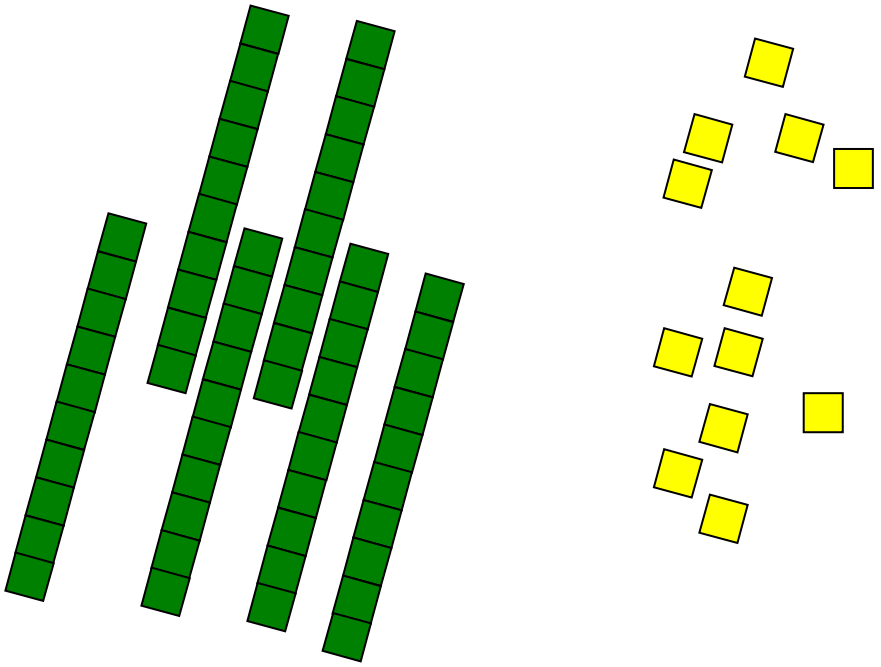




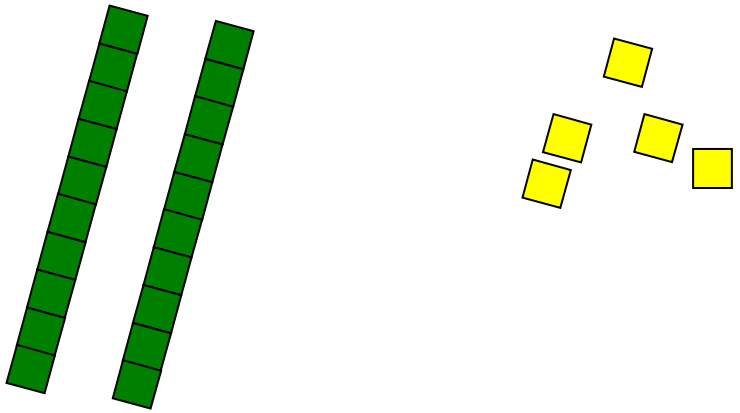
This is now “Sixty-twelve”

$6\,\not{7}^{1}\,2$

72 - 47

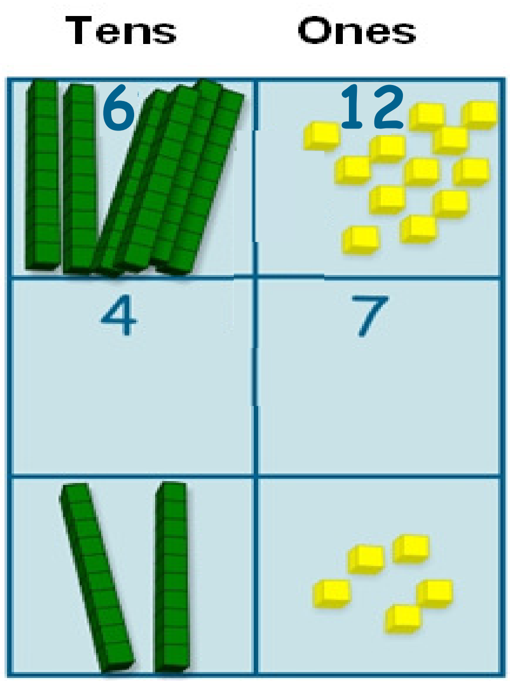


72 - 47 = 25

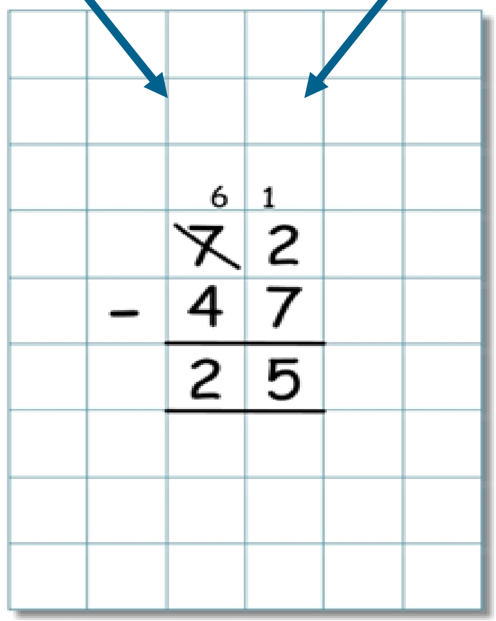


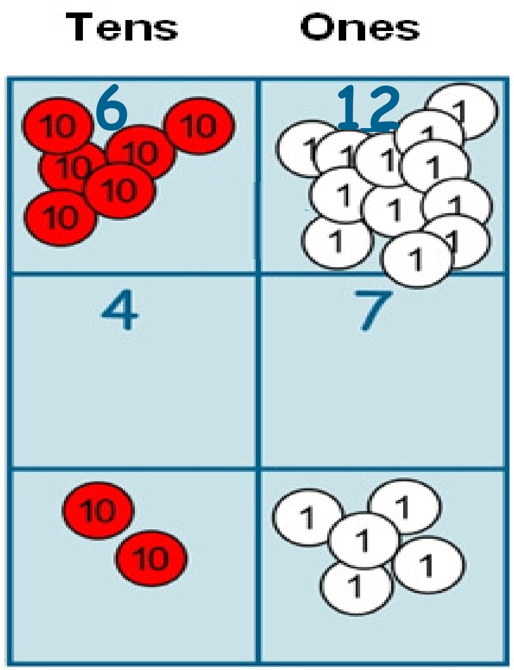
Really good for children in Year 2 to explore this through word problems making pictorial recordings and not formal algorithmic recording.

Compacted



leading to





Task

Explore some addition and subtraction calculations using the different manipulatives e.g.

– 158 + 167

– 234 - 145

- How well do the manipulatives help you to solve the calculation problems?
- How well do the manipulatives help to move pupils towards written methods?