Multiplication and Division:

concrete to visual to abstract

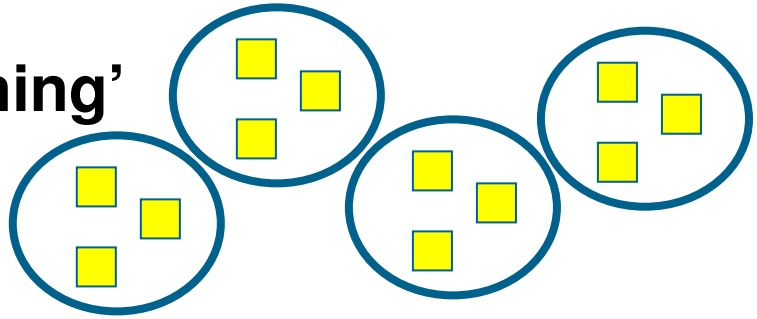
Multiplication

How would you solve these?

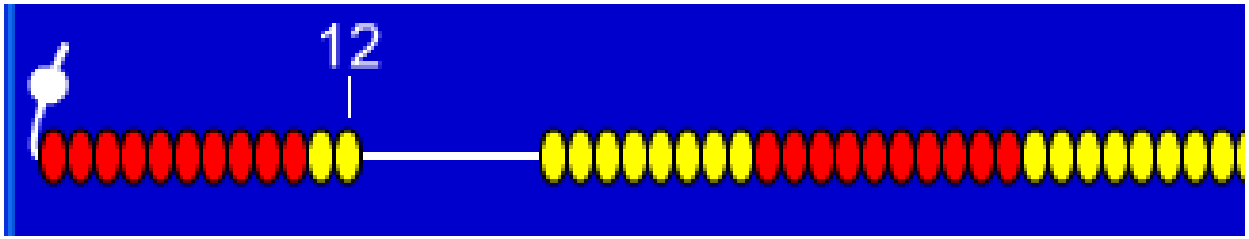
- 24×50
- 24×4
- 24×15
- 136×9
- 245×1.6

Models for multiplication

Lots of the 'same thing'



Bead Bar



Number Line

Fingers

"3" "6" "9" "12"

Models for multiplication

Partitioning:

24 x 5

20 x 5 = 100
4 x 5 = 20

100 + 20 = 120

```
  24
x  5
100
 20
120
```

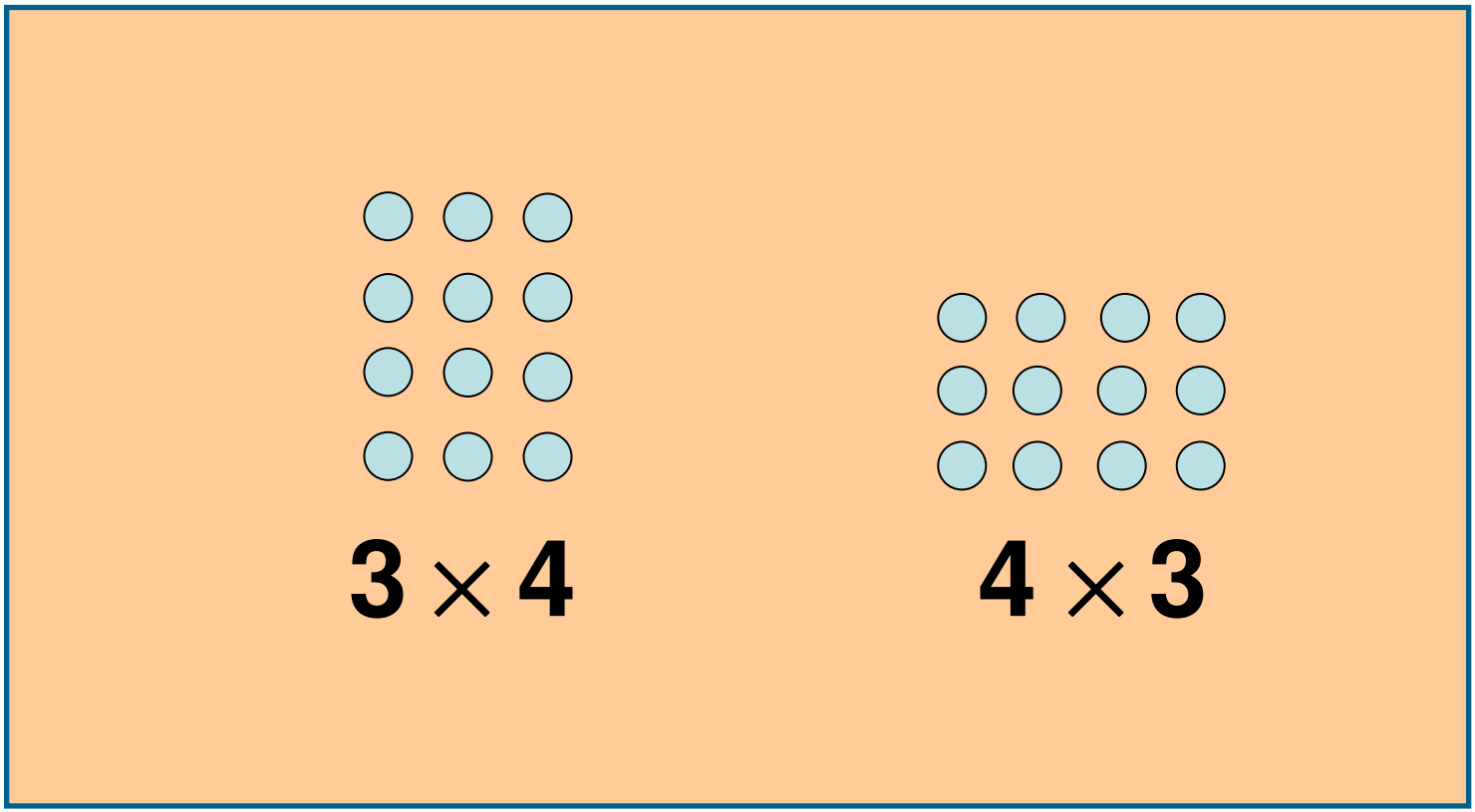
Grid method:

24 x 5

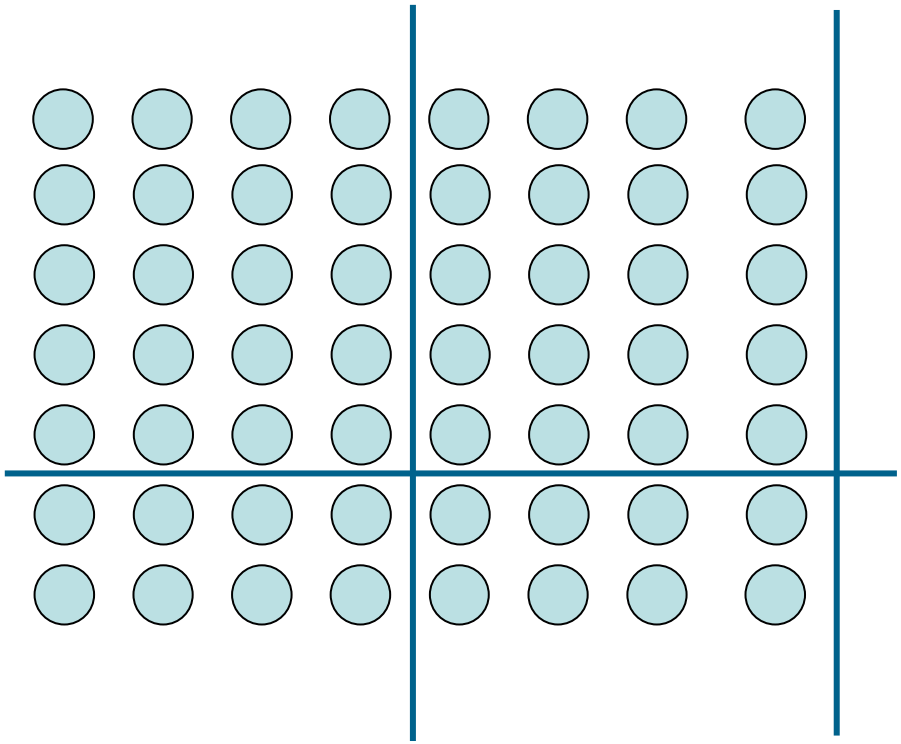
	20	4
x 5	100	20

100 + 20 = 120

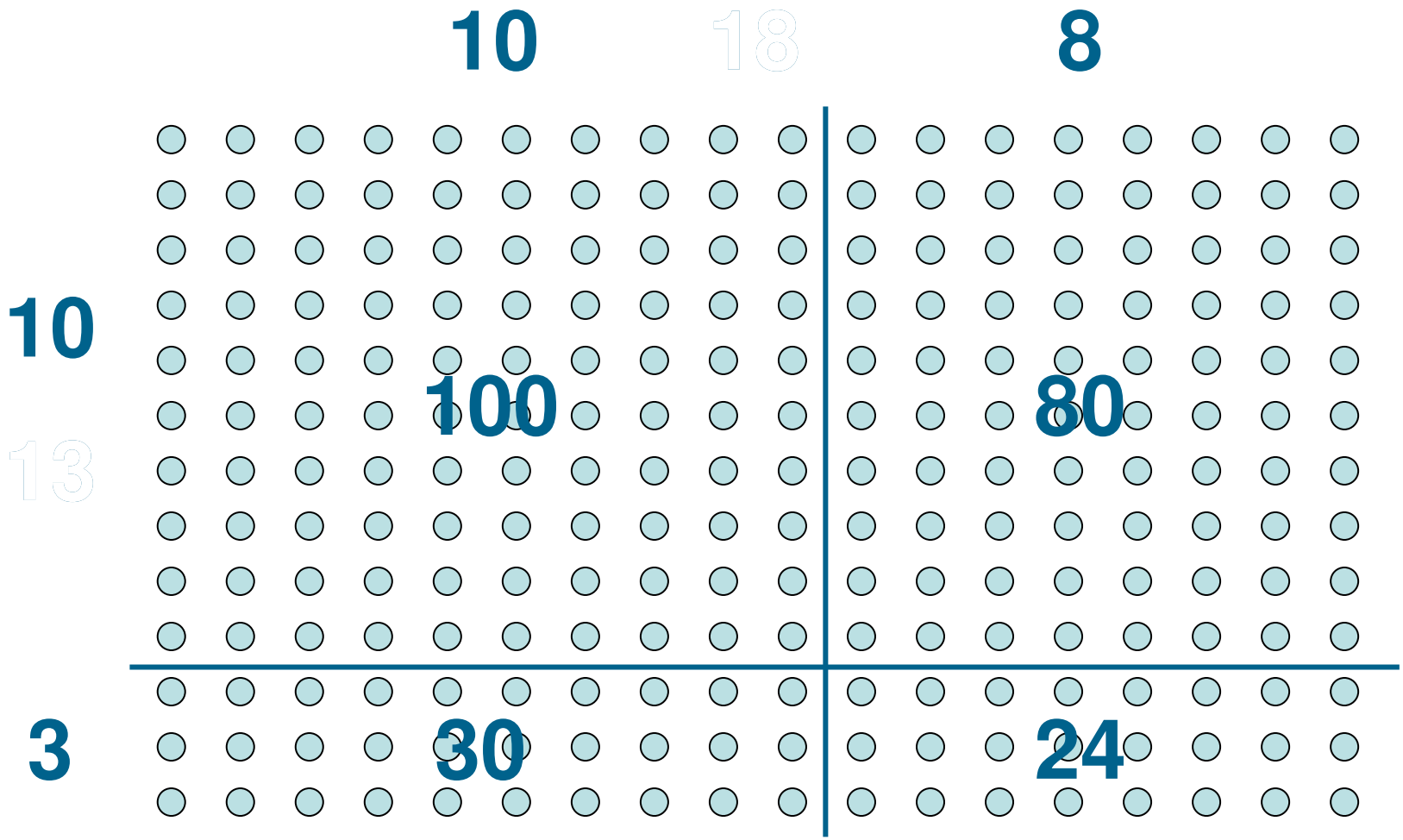
Models for multiplication



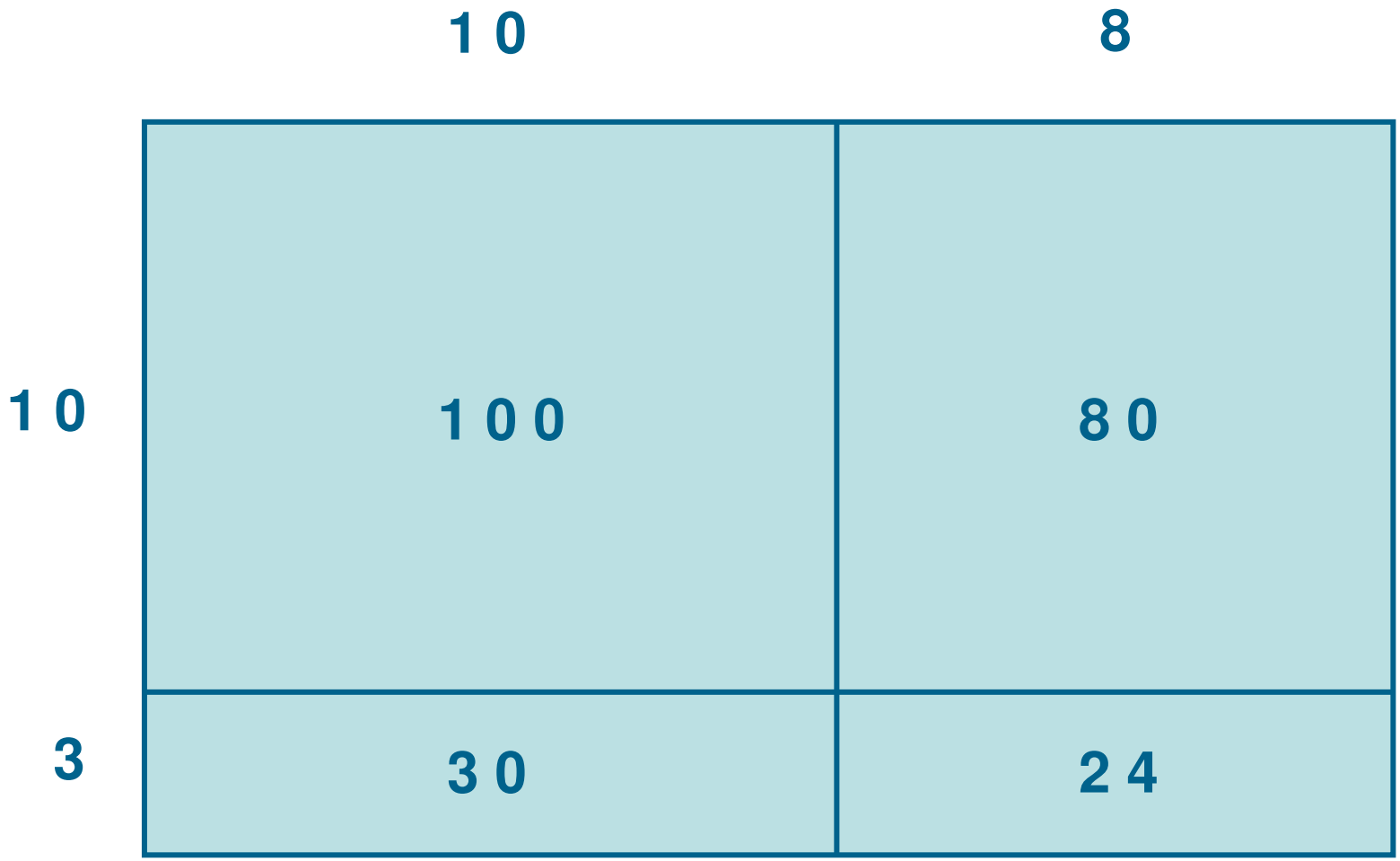
An image for $7 \times 8 = 56$

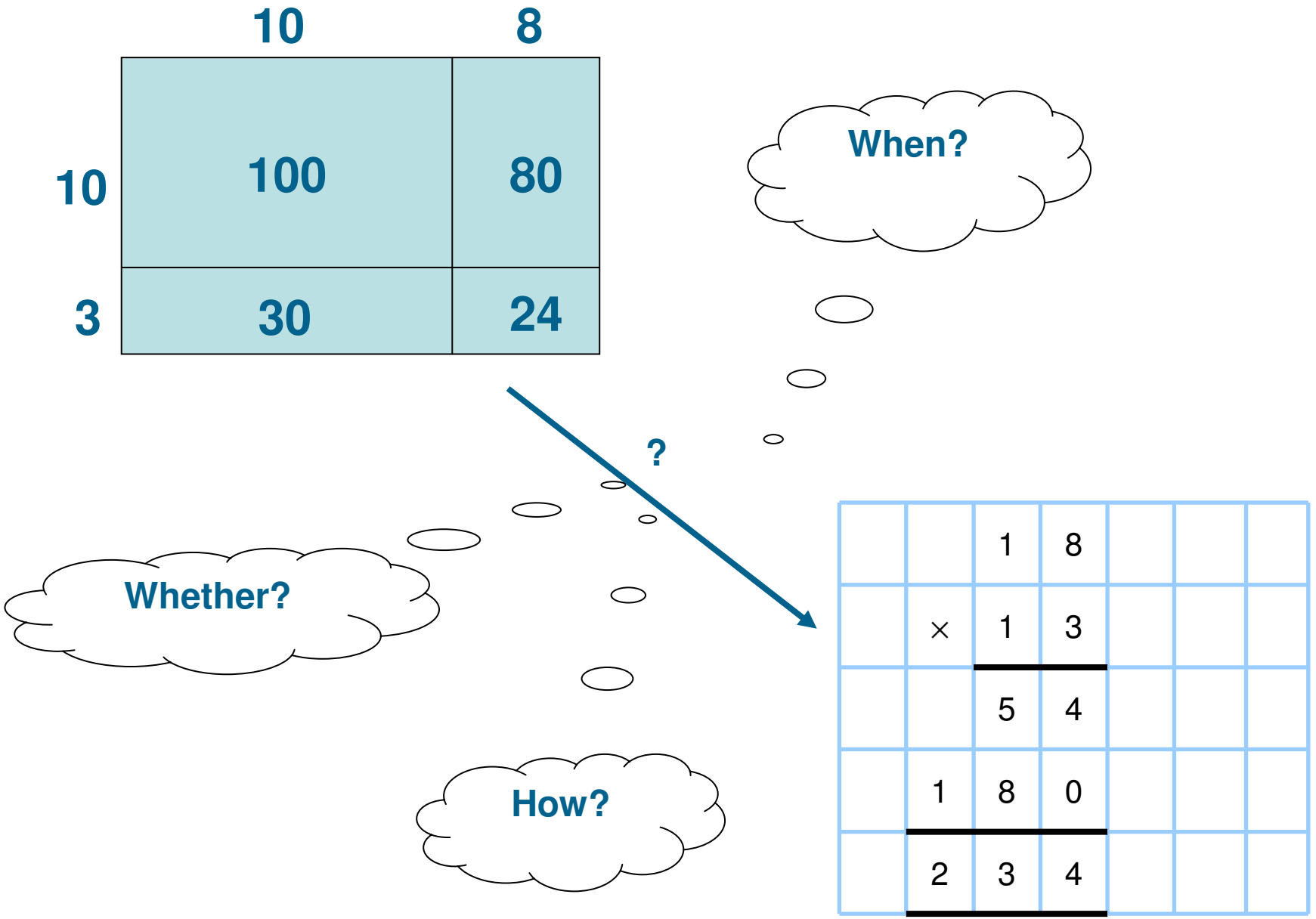


More than single digits?

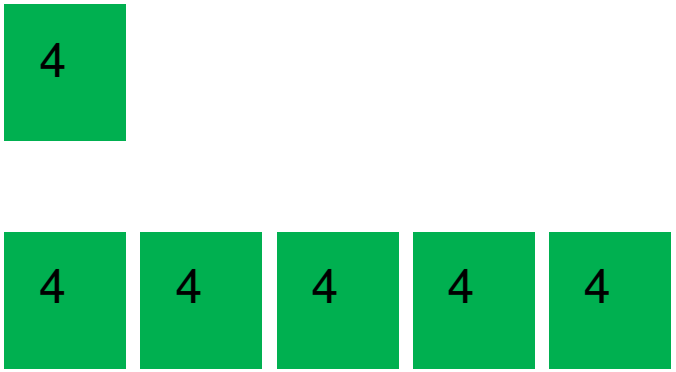


Progressing towards the standard algorithm




10
8
10
100
80
3
30
24
When?
?
Whether?
How?
1 8
× 1 3
5 4
1 8 0
2 3 4

- Peter has 4 books
- Harry has five times as many books as
- Peter. How many books has Harry?



- Henry ate 10 meatballs at the Christmas party. Shane ate 3 times as many meatballs as Henry . How many meatballs did they eat altogether?

- Helen has 9 times as many football cards as Sam. Together they have 150 cards. How many more cards does Helen have than Sam?

- The sum of 2 numbers is 60. One number is 9 times as big as the other. What is the bigger number?

- The sum of 2 numbers is 64. One number is 7 times as big as the other. What is the smaller number?

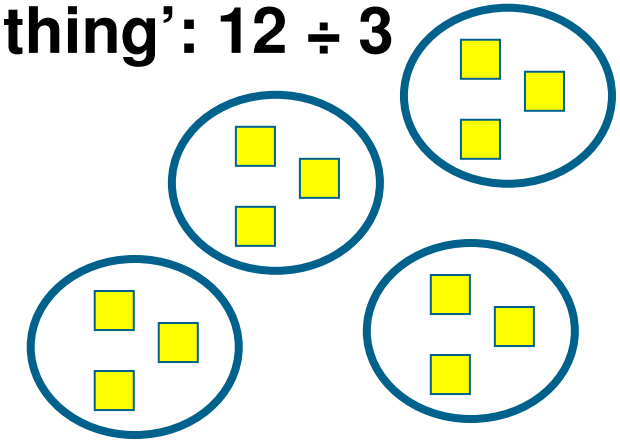
Division

How would you solve these?

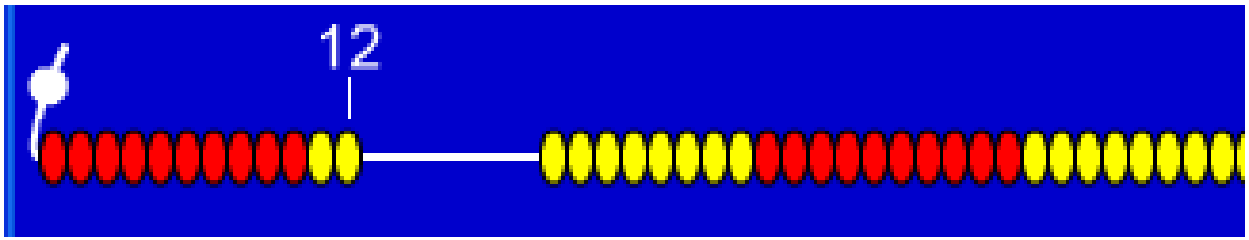
- 123 ÷ 3
- 165 ÷ 10
- 325 ÷ 25
- 408 ÷ 17
- 623 ÷ 24

How are these models for division as well as multiplication?

Groups of the ‘same thing’: 12 ÷ 3



Bead Bar



How are these models for division as well as multiplication?

Number Line

Fingers

“3 “6 “9 “12

Number line, counting in larger multiples: 122 ÷ 3

2 left, 2 out of a possible group of 3 which is 2/3

This can also be done vertically:

122 ÷ 3

120 (3 x 40)

2

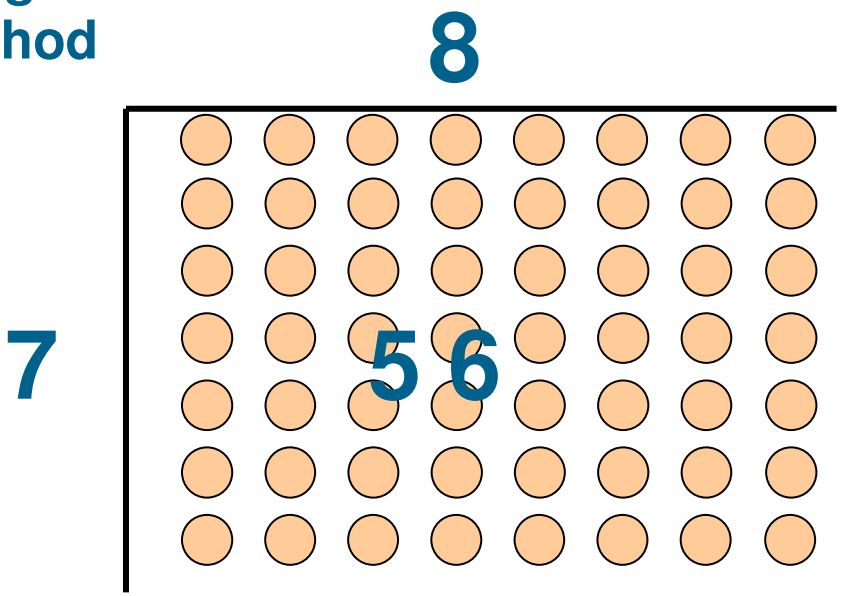
Or can be partitioned:

120 + 2

40 lots of 3 with remainder of 2

An image for 56 ÷ 7

An array is an image for division as well as multiplication and lends itself to the progression to the short method

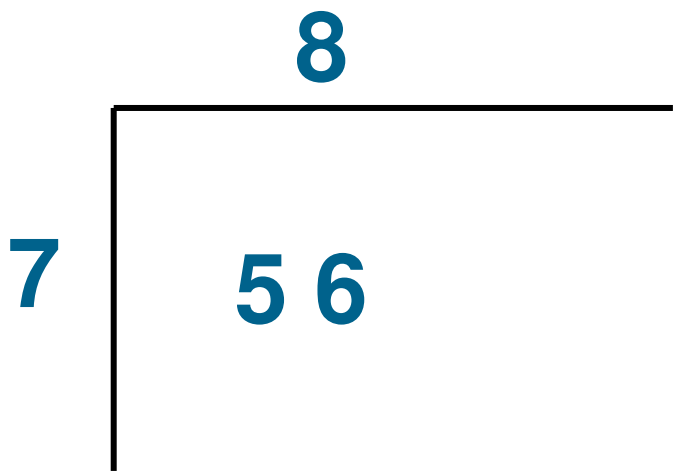


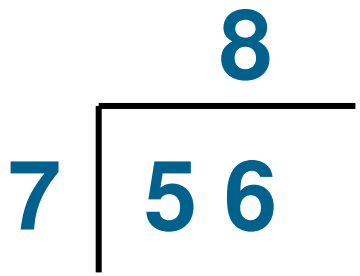
Either:

- How many 7s can I see? (grouping)

Or:

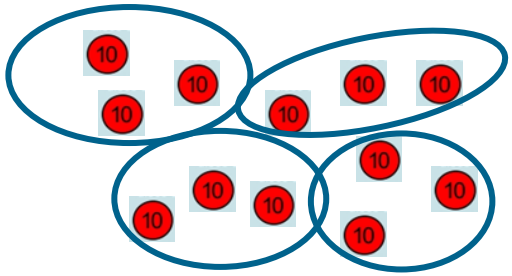
- If I put these into 7 groups how many in each group? (sharing)


8
7
5 6


8
7
5 6

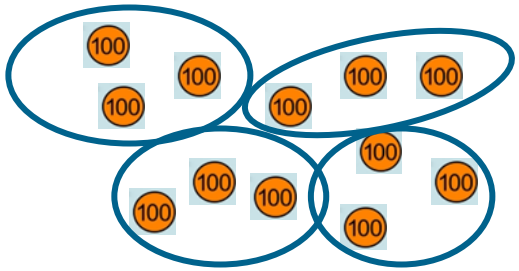
The power of the place value: counters for larger numbers

$$120 \div 3$$



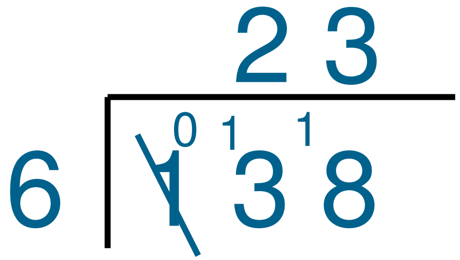
$$\begin{array}{r|l} & 40 \\ \hline 3 & 120 \end{array}$$

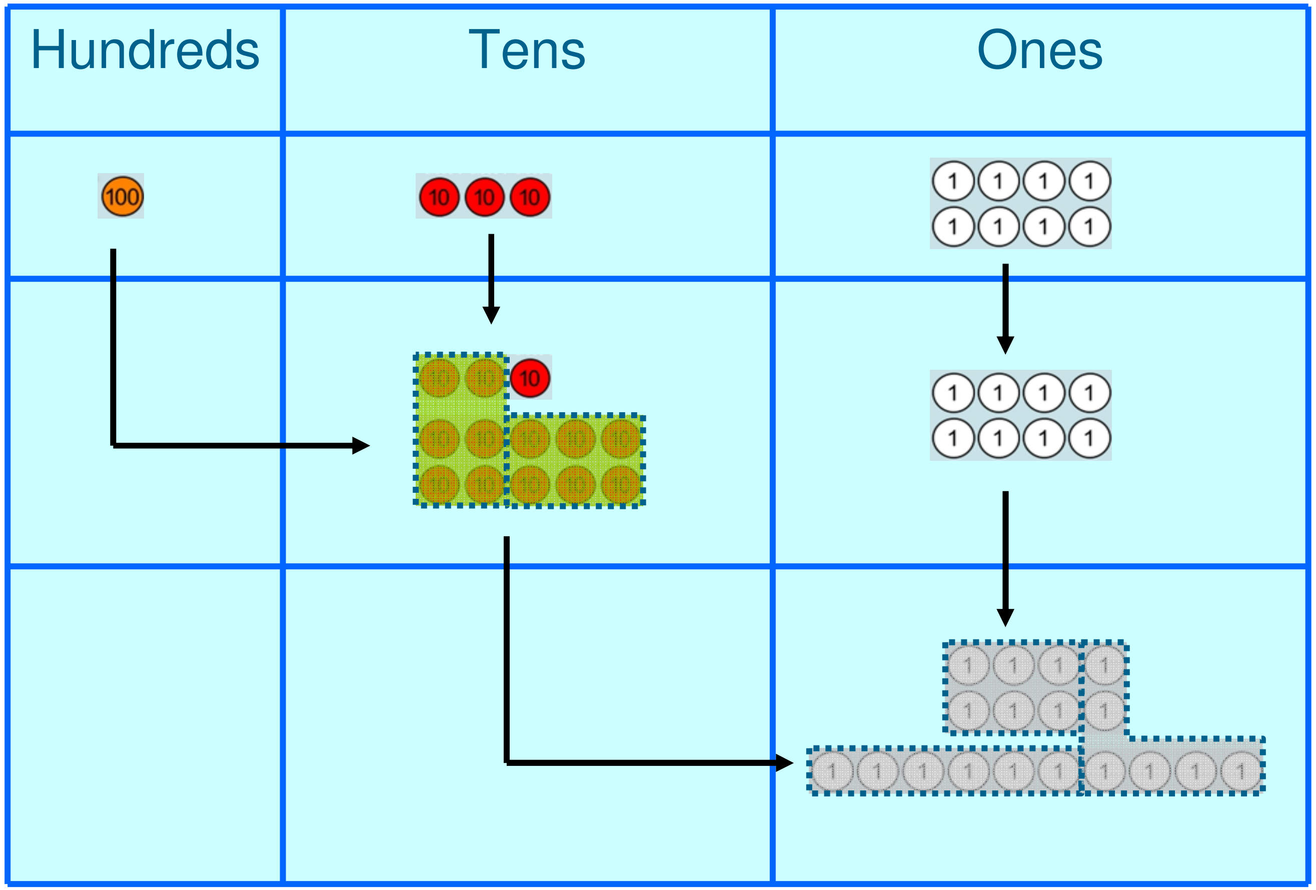
1200 ÷ 3

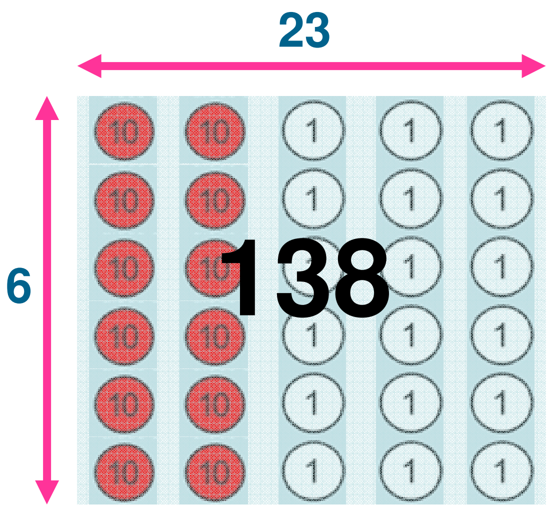


Similarly for 100s

$$
\begin{array}{r}
400 \\
3\overline{)\,1200} \\
\end{array}
$$


2 3
6 | 1 3 8
0 1 1


Hundreds
Tens
Ones


23
6
138

Try this with place value counters or Dienes:

- Make 120 using Dienes/place value counters
- Think of all the ways the number can be grouped
- Demonstrate each one that you think of

- 108 Year 3 children are going on a field trip to the art museum. Each bus must carry 12 children. How many buses are needed?

- Mr Smith had a piece of wood that measured 36 cm. He cut it into 6 equal pieces. How long was each piece?

- Sam had 5 times as many marbles as Tom. If Sam gives 26 marbles to Tom, the two friends will have exactly the same amount. How many marbles do they have altogether?

Task

Explore some division calculations using the different manipulatives.

- How well do the manipulatives help you to solve the calculation problems?
- How well do the manipulatives help to move pupils towards written methods?
- Reflect on your own practice about how a written method for division can be taught.

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144